Math 303 Test 2: November 13, 2001 C. Robinson

No books, no notes

Start each problem on a new page.

1. (25 Points) Consider the system of differential equations

$$\dot{x} = y$$
$$\dot{y} = -x - x^3.$$

   (a) Find the fixed points.
   (b) Find the potential function and graph it.
   (c) Draw the phase portrait.

2. (25 Points) Consider the system of differential equations

$$\dot{x} = y$$
$$\dot{y} = -x - x^3 - y\,(x^2 + y^2).$$

   Use a Lyapunov function to show that the origin is asymptotically stable. Explain your reasons for this to be true.

3. (25 Points) Consider the system of differential equations

$$\dot{x} = y$$
$$\dot{y} = -x - x^3 + \mu\,y - y\,(x^2 + y^2).$$

   (a) Show that there is an Andronov-Hopf bifurcation at $\mu = 0$. Is it subcritical or supercritical? Is the periodic orbit attracting or repelling?
   (b) What are the limit sets for $\mu > 0$ and $\mu < 0$?

4. (25 Points) Consider the system of differential equations

$$\dot{x} = y$$
$$\dot{y} = -\frac{x}{4} + 16\,y - y\,(x^2 + y^2).$$

   Show that the system has a periodic orbit.