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Methods of (micro)localization

Classical | **Quantum**

$$\begin{array}{c} \mathcal{X} \\ f \downarrow \\ X \end{array} \quad \rightsquigarrow$$

X affine (sing) sympl
- normal
- ω_X reg, $\tilde{X} \xrightarrow{\pi} X$

U_c filtered NC alg

$\operatorname{gr} U_c \simeq \mathbb{C}[X]$

Will use module cats...

Ex $\mathcal{X} = T^*(G/B)$

$f \downarrow$

$X = N(\mathfrak{g})$
nilp. cone

$$\begin{array}{c} D_{G/B}(c) - \mathrm{mod} \\ f^* \uparrow \downarrow f_* \\ U_c = U(\mathfrak{g})_c - \mathrm{mod} \end{array}$$

Beilinson-Bernstein localization

$$D_{G/B}(c) \rightleftarrows U(\mathfrak{g})_c - \mathrm{mod}$$

are derived equiv iff $c \notin -\rho + \left(\begin{array}{l}\text{union of}\\ \text{root}\\ \text{hyperplanes}\end{array}\right.$

Toy model $\quad V \underset{A^*}{\overset{A}{\rightleftarrows}} W$ Hermitian v.s.

Assume $A^* A = 1_V$

Q How can I tell they are isom?

A $\operatorname{Im}(A) \perp \ker(A^*)$

<u>Isom</u> if W can't be split as nontriv
$\operatorname{Im}(A) \oplus \ker(A^*)$

This works very well for categories!

<u>Setting</u> $\mathcal{C} \underset{Rf_*}{\overset{Lf^*}{\rightleftarrows}} \mathcal{D}$ triang cats

<u>Prop</u> [cf B, BK, BMR, ...]

(1) Suppose $\mathcal{D}$ cptly gen, Rf_* preserves colim, takes set of cpt gens to cpt objs
Then Rf_* has a right adj $f^!$ that is continuous

(2) If, in addition, $\exists$ set S' of compact generators of $\mathcal{C}$ for which

$\mathbb{L}f^* U = f^! U \quad \forall U \in S'$,

<u>and</u> $\mathbb{1}_{\mathcal{C}} \to Rf_* \circ \mathbb{L}f^*$ is an

equivalence, <u>and</u> if $\mathcal{D}$ is indecomposable, <u>then</u> $\mathbb{L}f^*$, Rf_* are mutually quasi-inverse equivalences

Application to Quantum Setting (joint w K. McGerty)

Meta-Theorem

$$D(U_c\text{-mod}) \underset{\mathbb{R}f_*}{\overset{\mathbb{L}f^*}{\rightleftarrows}} D(\mathcal{E}_{\mathfrak{X}}(c)\text{-mod})$$

TFAE (1) $\mathbb{L}f^*, \mathbb{R}f_*$ are mutually quasi-equiv

(2) $\mathbb{L}f^*$ is coh bounded

(3) U_c has finite global dimension

Smooth affine symplectic variety

$$\begin{array}{c} \mathfrak{X} = \mu^{-1}(0) /\!/_{\chi} G \\ \big\downarrow \\ X = \operatorname{Spec}\left(\mathbb{C}[W] \big/ \mu^*(\mathfrak{g}) \right)^G \end{array}$$

$\chi : G \to \mathbb{C}$ group character

$$\mathfrak{g} \xrightarrow{\mu^*} \mathbb{C}[W]$$

~~Because~~ $GL_n \circlearrowright T^*(\mathfrak{gl}_n \times \mathbb{C}^n)$

$$\begin{array}{c} \mathcal{X} = (\mathbb{C}^2)^{[n]} \\ \downarrow \\ X = S^n\mathbb{C}^2 \end{array}$$