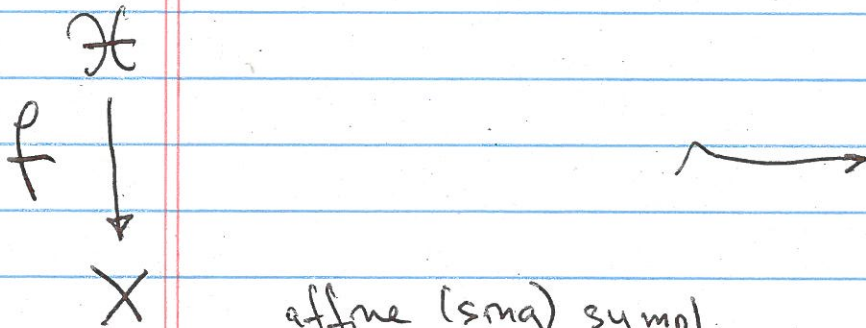


Methods of (micro)localization

Classical

Quantum



affine (smg) sympl

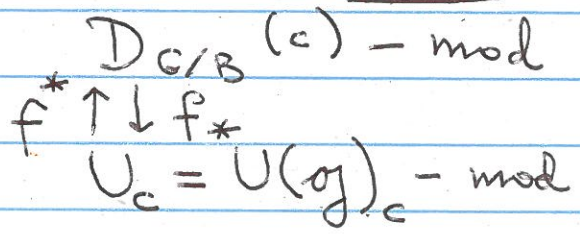
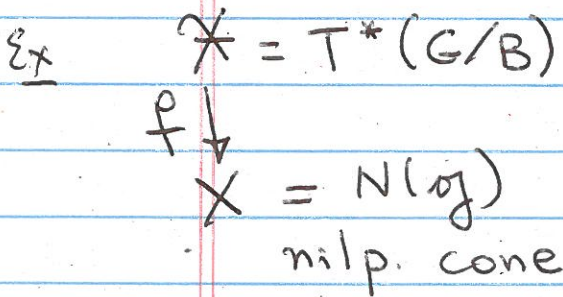
• normal

•  $w_x$  reg,  $\tilde{X} \xrightarrow{\mathbb{P}} X$

$U_c$  filtered NC alg

gr  $U_c \simeq \mathbb{C}[X]$

Will use module cats...

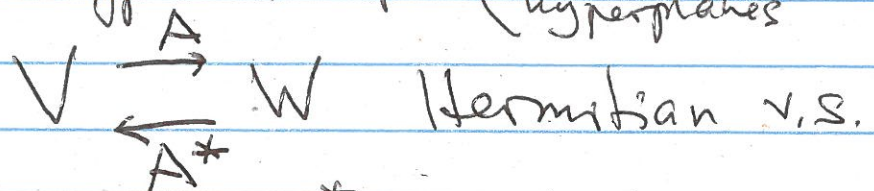


Beilinson-Bernstein localization



are derived equiv iff  $c \notin -\rho + \left( \begin{array}{l} \text{union of} \\ \text{root} \\ \text{hyperplanes} \end{array} \right)$

Toy model



Assume  $A^* A = 1_V$

Q How can I tell they are isom?

A  $\text{Im}(A) \perp \text{Ker}(A^*)$

Isom if  $W$  can't be split as nontriv  
 $\text{Im}(A) \oplus \text{Ker}(A^*)$

This works very well for categories!

Setting  $\mathcal{C} \xrightarrow{Lf^*} \mathcal{D}$  triang cats  
 $\mathcal{D} \xleftarrow{Rf_*} \mathcal{C}$

Prop [cf B, BK, BMR, ...]

(1) Suppose  $\mathcal{D}$  cply gen,  $Rf_*$  preserves cplim, takes set of cpt gens to cpt objs then  $Rf_*$  has a right adj  $f^!$  that is continuous

(2) If, in addition,  $\exists$  set  $S'$  of compact generators of  $\mathcal{C}$  for which

$$Lf^* U \cong f^! U \quad \forall U \in S'$$

and  $\mathbb{1}_{\mathcal{C}} \rightarrow Rf_* \circ Lf^*$  is an

equivalence, and if  $\mathcal{D}$  is indecomposable, then  $Lf^*, Rf_*$  are mutually quasi-inverse equivalences



# Application to Quantum Setting (joint w K. McGerty)

## Meta-Theorem

$$\mathcal{D}(U_c\text{-mod}) \begin{array}{c} \xrightarrow{L_f^*} \\ \xleftarrow{Rf_*} \end{array} \mathcal{D}(\mathcal{E}_X(e)\text{-mod})$$

- TFAE
- (1)  $L_f^*$ ,  $Rf_*$  are mutually quasi-equiv
  - (2)  $L_f^*$  is coh bounded
  - (3)  $U_c$  has finite global dimension

Smooth affine symplectic variety

$$X = \mu^{-1}(0) // \chi G \quad \chi: G \rightarrow \mathbb{C}$$

group character

$$\downarrow$$
$$X = \text{Spec} \left( \mathbb{C}[W] / \mu^*(\mathfrak{g}) \right)^G$$
$$\mathfrak{g} \xrightarrow{\mu^*} \mathbb{C}[W]$$

~~Because~~  $GL_n \mathbb{C} \cong T^*(\mathbb{C}P^1 \times \mathbb{C}P^1)$   $\mathcal{E} = (\mathbb{C}P^2)^{[n]}$

$\downarrow$   
 $X = S^n \mathbb{C}P^2$