Base	Cat	D-mod	Explicit
M	$\mathrm{DCoh}\,M$	$HP_\bullet^{\mathcal{O}_M}(\mathrm{DCoh}(M)) \cong \omega_M((u))$	$\Omega_M^{\cdots}[\ldots]$
	$\mathrm{Perf}(M_0) \to \mathrm{DCoh}\,M_0 \to MF(M,f)$	$RH^{-1}(\underline{\mathbb{C}}_{M_0}) \to \omega_{M_0} \to$ done	$[M_0]_{virt}$
pt	pushforwards of all		$R\Gamma(M, (\Omega^\bullet \otimes -, d))$
$\mathbb{A}^1_x$	$\mathrm{DCoh}(M)$	$HP_\bullet^{k[x]}(\mathrm{DCoh}\,M) \cong f_+\omega_M$	$\Omega_M^\bullet[\partial_x]$, $d + \partial_x(df\wedge)$, $+\partial f$
$\check{\mathbb{A}}^1_\tau \cup \hat{\mathbb{D}}^\circ(\tau=\infty)$, $\lambda = 1/\tau$	?	$\mathrm{Four}(f_+\omega_M)$	$\Omega_M^\bullet[\tau]$, $d + \tau df$, $\partial_\tau = f$
		$\mathrm{Four}(f_+\omega_M) \subset \Omega_M^\bullet \otimes k((\lambda))$,	$\lambda d + \mp df$, $\partial_\lambda = -f/\lambda$

$\check{\mathbb{A}}^1[+2]_\beta$ $\qquad MF(M, f) \qquad HP_\bullet^{/k((\beta))}(MF) \simeq \Omega_M^\bullet((u/\beta)), \tfrac{u}{\beta}d + df$

$\mathrm{Spec}\, k[[\beta]]$ $\qquad \displaystyle\bigoplus_{\substack{c \\ \in \\ \text{crit val}(f)}} R\Gamma(\Phi_{f-c}) \otimes e^{\lambda c}$

rank one irregular $\mathcal{D}$-mod on the λ disk

(Sabbah, Twisted De Rham 2)

$\cong$

E_2-alg, B_∞-alg, A_∞-alg

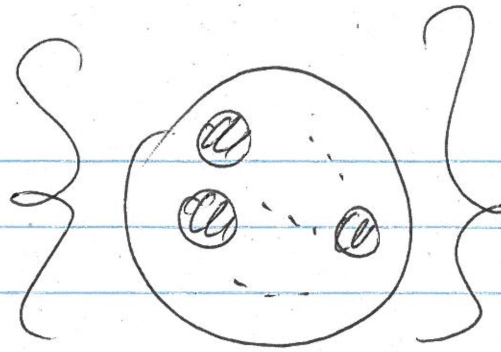
<u>Dfn</u> A B_∞-alg A is an alg in A_∞ algs.

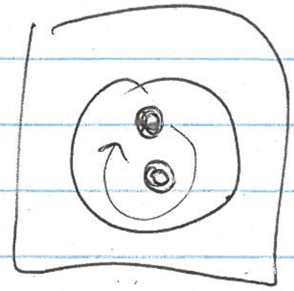
<u>Dfn</u> An A_∞ alg A is the data of a dg coalgebra structure on

$$\mathcal{B}(A) = \bigoplus_{n \geq 0} (A[1])^{\otimes n}$$

with the usual cofree comult.

a dg <u>bi</u>algebra structure...

$E_2(n) =$ 

$E_2(2)$ 

$\wr$ h.e.

S^1

At the level of B_∞ algebras:

$$\mathrm{Prim}\,(B\ell A) = A[+1]$$

<u>Question</u> $\mathcal{C}$ - k-linear dg category.
<u>What</u> does it take to make it $k[x]$-linear?

<u>Answer</u>

$$E_2\text{- map} \quad k[x] \longrightarrow HH^\bullet(\mathcal{C})$$

$$\Updownarrow$$

$$\text{Lie-map} \quad k[+1] \longrightarrow HH^\bullet(\mathcal{C})[+1]$$

<u>Prop</u> $U_{E_2}(k[1]) = k[x]$

Lie (or rather L_∞) map $k[+1] \to HH^\bullet(\mathcal{C})[+1]$

$\Updownarrow$

$MC\left(\mathfrak{m}_{k[[\beta]]} \hat{\otimes} HH^\bullet(\mathcal{C})[+1]\right)$

$\shortparallel$

$\left\{\text{Curved } \mathcal{C}^{k[[\beta]]\text{-lin}} \text{ def of}\right\}$

$k[x]$-lin dg cat $\simeq$ dg cat acted upon by $k[+1]$

<u>Construction</u> $\mathcal{C}_{k[+1]} \to \mathcal{C}^{k[+1]}$

coinv invariants

<u>Ex</u> $DCoh(M)$ as $k[x]$-lin

$(\_\_\_)^{k[+1]} \simeq DCoh(M_0)$

$(\quad)^{Tate} \simeq MF(\ldots)$

<u>So what?</u> $k[+1] \circlearrowright HH^\bullet(\mathcal{C})$ $HH_\bullet(\mathcal{C})$

$HP_\bullet(\mathcal{C})$.

etc.

and can ask to compare $HH^\bullet(\mathcal{C}^{Tate})$ to $HH^\bullet(\mathcal{C})^{Tate}$

2) The construction

$$(\star) \quad \mathcal{C}/_{k[x]} \rightsquigarrow \mathcal{C}^{Tate}/k((\beta))$$

wants to categorify eg Fourier transform of $\mathcal{D}$-modules.

Variant $\quad M \;—\; \mathcal{C}$ smooth and proper$/\mathcal{O}_M$

$$\begin{tikzcd} M \arrow[d, "f"'] \arrow[dr, "\Gamma_f"] & \\ \mathbb{A}^1 & M \times \mathbb{A}^1 \end{tikzcd}$$

$\Gamma_{f*}\mathcal{C} \quad$ lin$/\mathcal{O}_{M\times\mathbb{A}^1}$

$$\mathrm{Four}^{(0,\infty)}_{/M}(\Gamma_{f*}\mathcal{F}) \in D^{rh}_{\hat{D}^*(\infty)}(D(M))$$

$$D^{rh}_{\hat{D}^*(\infty)}(D(M)) \cong k[\mathbb{Z}]\text{-mod}(D(M))$$

by Sabbah...

$\Phi_f(\mathcal{F})$

So: For $\mathcal{C}$ smooth, proper$/\mathcal{O}_M$

there is a version of $\star$:

can do $(\Gamma_{f*}\mathcal{C})^{Tate/M}$ linear over $\mathcal{O}_M((\beta))$