C. Roucairol (2006-7). 1) To detect the irreg sing of N on $\mathbb{A}^1$

2) To compute the formal structure (decomp) at these pts

3) to compute Stokes filtr

$\mathbb{C}$ irreg sing $\Rightarrow f^{-1}(c)$ is asympt to S

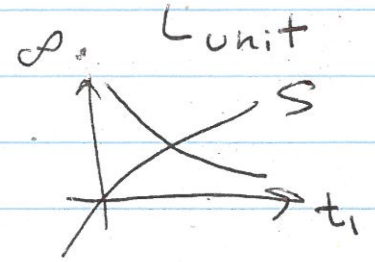
$$\begin{tikzcd} & e^{\varphi(t_1)} \\ {} \arrow[ur] \arrow[dr] & \\ & (R_\varphi, \nabla) \end{tikzcd}$$

$\leftarrow$ monodromy operator ~~on~~ T_φ

a) Possible φ are given by Puiseau expansion for each ~~graph~~ branch

(b) in good cases, T_φ is that of the loc system $\Phi(DR\,M)$

$$t_2 = (t_1 - c)^{-\frac{q}{p}} \underbrace{u}_{\text{unit}}((t_1 - c))$$



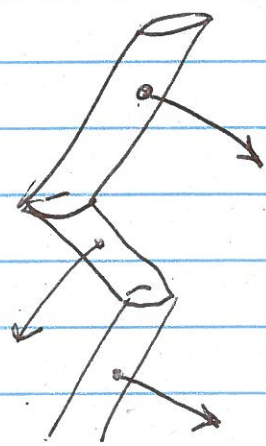
Stokes filtered loc syst
to resolve sing at (c, ∞)

$$\tilde{X}(D) \to \tilde{A}^1(c)$$

$$\partial\tilde{X}(D) \to S^1$$

$$\partial\tilde{X}(D) \longrightarrow \theta_0$$
$$\theta_0$$

like a pipe around the components of $\tilde{f}^{-1}(c)$



branches coming to the pipe and make some holes.

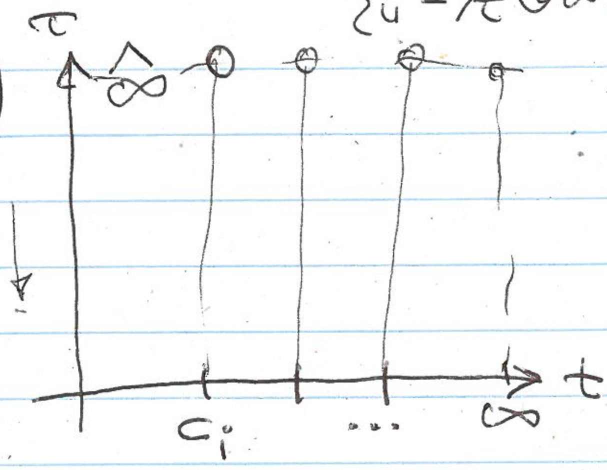
(leaky pipe)

$$\mathcal{L}_{\leq \varphi, \theta}$$

Stokes filtration for the Fourier transform of a regular holonomic module / $\mathbb{C}[t]\langle\partial_t\rangle$-

$$\underset{\underset{{}^{F}M}{\shortparallel}}{N} = \hat{\pi}_+\left(\pi^+ M \otimes e^{t\tau}\right)$$

$\begin{cases} z \\ u \end{cases} = 1/\tau \ @ \ \hat{\infty}$



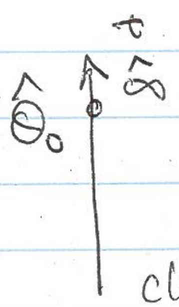
Possible φ are the c_i/t

<u>To compute</u> $\mathcal{L}_{\leq c_i/t,\ \hat{\theta}_0}$

Need to blow up the point $(c_i, \hat{\infty})$ only once. Want that

$$e^{(t-c_i)\tau}$$

has a well-defined behavior

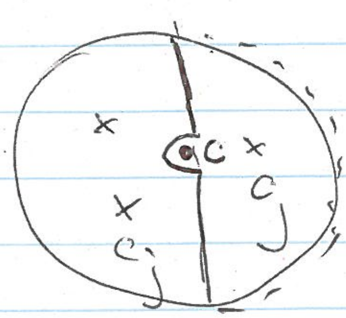


closed $= \widetilde{\mathbb{P}}^1$

disc

$$\mathcal{L}_{\hat{\theta}_0} = \beta_! \, \alpha_* \, \mathcal{F}$$

$$\mathcal{F} = \mathcal{D}RM \text{ on } \mathbb{A}^1 = \mathring{\Delta}$$



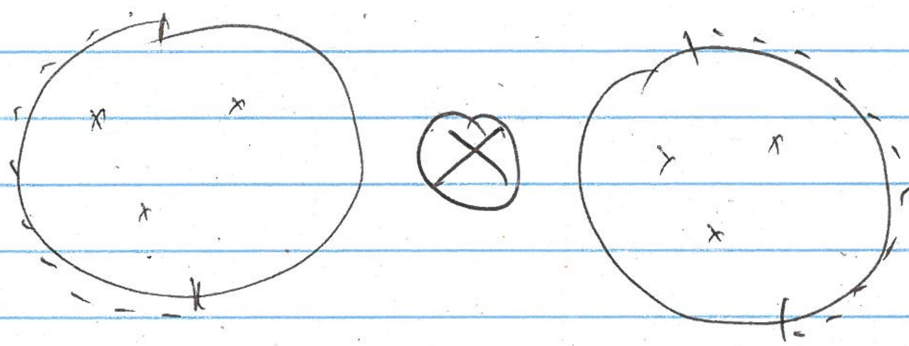
$$\begin{array}{l} \alpha : \mathring{\Delta} \hookrightarrow \mathring{\Delta} \cup \text{ (open)} \\ \qquad \qquad \downarrow \beta \text{ (close)} \\ \qquad \qquad \Delta \end{array}$$

$$\mathcal{L}_{\leq c/t,\, \hat{\theta}_0} = R\Gamma(\beta_{c!}\, \alpha_{c*} \mathcal{F})$$

$$\mathcal{L}_{< c/t,\, \hat{\theta}_0} = R\Gamma(\beta_{c!}\, \alpha_{c*} \mathcal{F})$$

<u>Duality</u>

$$R\Gamma \tilde{\beta}_{c!}\, \tilde{\alpha}_{c*} \mathcal{F} \otimes R\Gamma \beta_{c!}\, \alpha_{c*} \mathcal{F}$$



$$\to R\Gamma_c(\mathbb{C}) = \mathbb{C}$$

? $\mathcal{F}$ defined $/\mathbb{Q}$ $\Rightarrow$ everything (?) defined over $\mathbb{Q}$ ...

As in nc Hodge structures...

Fourier ~~T~~ transform of a regular holonomic $\mathcal{D}$-module is an nc Hodge structure (<u>pure</u>).

? The method used here is the only way to prove OPPOSEDNESS.