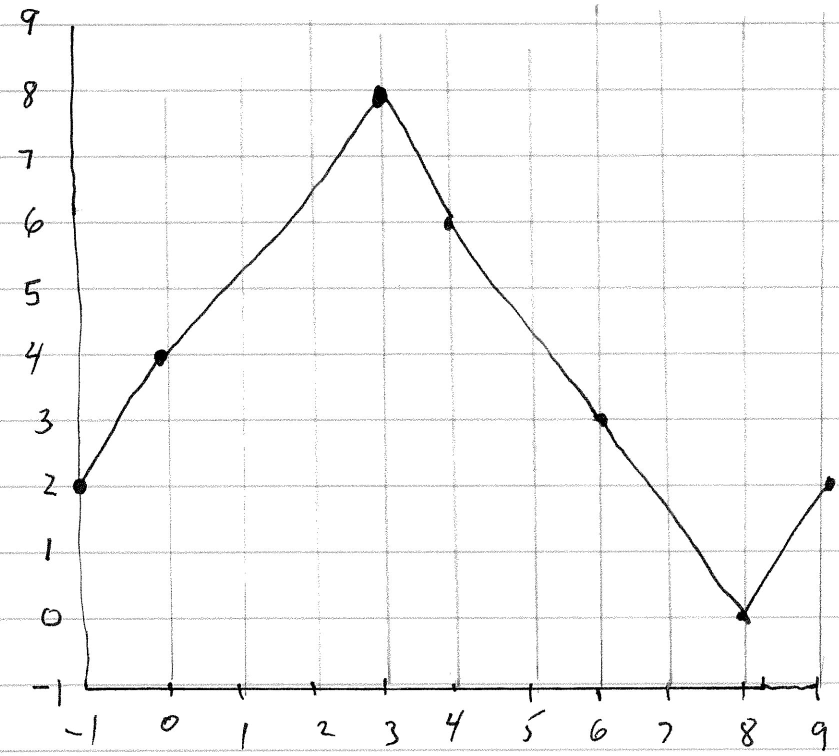
Final 2002

5

(a)



(b) $f[0,3] = [4,8] = [4,6] \cup [6,8]$

$f[3,4] = [6,8] = [6,8]$

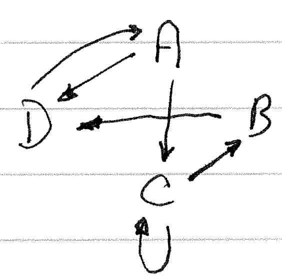
$f[4,6] = [3,6] = [3,4] \cup [4,6]$

$f[6,8] = [0,3] = [0,3]$.

Markov partition $\{[0,3], [3,4], [4,6], [6,8]\}$

Intervals cover combination of intervals as given above. $[0,3] = I_A$ $[3,4] = I_B$ $[4,6] = I_C$ $[6,8] = I_D$.

(c)



Transition graph.

n	Symbol	Periodic
1	C^∞	Yes
2	$(AD)^\infty$	Yes
3		No
4	$(CBDA)^\infty$	Yes
5	$(CCBDA)^\infty$	Yes

all periods except 3

(e) $F[-1,9] = [0,8] \subset (-1,9)$.
contained in interior. so $[-1,9]$ trapping region
$F[0,8] = [0,8]$ so invariant.
~~so~~ $[0,8]$ is the attracting set for
the trapping region $[-1,9]$

(f) & (h) Transition graph is irreducible.
There are two choice to go from C and
two choices to go from A.
~~Therefore f~~ f continuous piecewise expanding
map with Markov partition.
Therefore, topologically transitive on $[0,8]$
and has sensitive dependence.

(g)
At each point $|f'(x)| = \frac{4}{3}$, 2, or $3/2$.
The Lyapunov ~~also~~ exponent is an average
of $\ln(4/3)$, $\ln(2)$, and $\ln(3/2)$.
Therefore
$$0 < \ln(4/3) \leq \ell(x_0, f) \leq \ln(2)$$
is positive.

(i) $[0,8]$ is an attracting set with sensitive
dependence & topologically transitive, so
it is a chaotic attractor.