Math 313-2 Final June 2002

① Let $\varepsilon_n = \frac{1}{n} - \frac{1}{n+1} = \frac{n+1-n}{n(n+1)} = \frac{1}{n(n+1)}$.

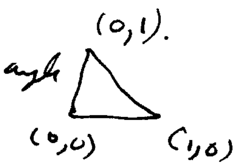
It takes
$$\frac{\frac{1}{n}}{\varepsilon_n} = n+1 \text{ intervals to cover } [0, \tfrac{1}{n}].$$

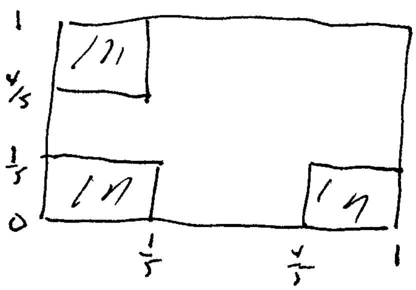
It takes $n-1$ intervals to cover $\{1, \frac{1}{2}, \cdots, \frac{1}{n-1}\}$.

$\therefore N(\varepsilon_n) = (n+1) + (n-1) = 2n$.

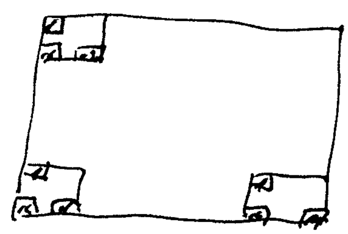
$$\lim_{n\to\infty} \frac{\ln N(\varepsilon_n)}{\ln(\varepsilon_n^{-1})} = \lim_{n\to\infty} \frac{\ln(2n)}{\ln(n)(n+1)} = \lim_{n\to\infty} \frac{\ln 2 + \ln n}{\ln n + \ln n+1}$$
$$= \tfrac{1}{2}.$$

② (a) Use images of either unit square or triangle (These will give the same attractor).





Then



Continuing, we get something like Sierpinski carpet, but with only 3 squares at each stage. Limit is a type of Cantor set.

(b) There are $(3)^3 = 27$ boxes of size $(0.2)^3 = 0.008$ needed to cover the attractor.

(c) Need $N(0.2^n) = 3^n$ of size 0.2^n.

$$\lim_{n\to\infty} \frac{\ln 3^n}{\ln 5^n} = \frac{\ln 3}{\ln 5} \text{ is box dimension.}$$