Math 353 **Midterm Test: May 7, 2010** **C. Robinson**

No books, no notes. Calculators are allowed
Show all your work in your bluebook. Start each problem on a new page.

1. (18 Points) Consider the scalar differential equation $\dot{x} = 4x - x^3$.

a. *Find* the fixed points and *classify* their stability type.

b. Draw the phase portrait.

2. (22 Points) Consider the following system of linear differential equations:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} 3 & \text{-}2 \\ 2 & \text{-}2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}.$$

a. *Give* the general real solution.

b. *Draw* the phase portrait. Be sure to indicate the direction of the trajectories.

3. (26 Points) Consider the following system of nonlinear differential equations:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} y - x^3 \\ x - y \end{pmatrix}.$$

a. *Find* the fixed points. Classify each of them as a stable node, stable focus, saddle, unstable node, etc.

b. Draw the phase portrait using the nullclines and the answer to part (a).

4. (34 Points) Consider the following system of nonlinear differential equations:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} y \\ x - x^3 - by \end{pmatrix}.$$

a. Sketch the graph of the potential function $V(x)$ for $b = 0$.

b. Draw the phase portrait of the system when $b = 0$.

c. Find a Lyapunov function for $b > 0$. Verify that it is a Lyapunov function.

d. Sketch the phase portrait of the system when $b > 0$.