Math 313-1 Test 1: January 29, 2002 C. Robinson

No books, no notes. You may use hand calculators

1. (20 Points) Consider the "tripling map" defined by

$$f(x) = 3\,x \qquad (\text{mod } 1).$$

   Determine the complete orbit of the points $1/8$ and $1/72$. Indicate whether each of these points is periodic, eventually periodic, or neither.

2. (20 Points) Let $f(x)$ be a function from the line $\mathbb{R}$ to itself. Suppose that, for every $k \geq 1$, f^k has $(3^k - 2^k)$ fixed points, e.g., there are $(3 - 2) = 1$ points fixed by $f = f^1$ and $(9 - 4) = 5$ points fixed by f^2. Make a table for $1 \leq k \leq 4$ showing the following: (i) k, (ii) number of fixed points of f^k, (iii) how many of these points fixed by f^k have lower period, (iv) number of points of period k, and (v) number of orbits of period k.

3. Consider the function

$$f(x) = \frac{5}{4}\,x - x^3.$$

   (a) (20 Points) Find the fixed points and classify each of them as attracting, repelling, or neither.

   (b) (20 Points) Use the cobweb plot analysis to determine the dynamic behavior of all the points with $-0.6 \leq x \leq 0.6$. Describe the orbits of representative points using words as well as by the plot. Hint: $f'(\pm\sqrt{\frac{5}{12}}) = 0$, $\sqrt{\frac{5}{12}} \approx 0.6455$, and $f'(x) > 0$ for $-\sqrt{\frac{5}{12}} < x < \sqrt{\frac{5}{12}}$.

4. (20 Points) Consider the "saw map", $S(x)$, defined by

$$S(x) = \begin{cases} 3\,x & \text{if } 0 \leq x \leq 1/3 \\ 2 - 3\,x & \text{if } 1/3 \leq x \leq 2/3 \\ 3\,x - 2 & \text{if } 2/3 \leq x \leq 1. \end{cases}$$

   See the figure for the graph of S. Use the three symbols L, C, and R, with corresponding intervals $I_L = [0, 1/3]$, $I_C = [1/3, 2/3]$, and $I_R = [2/3, 1]$. Give the order in the line of the nine intervals that correspond to strings of 2 symbols, e.g., I_{CR}.