Northwestern University

Name: _____

Id #:

Math 214-1 Common Final

Fall Quarter 2002

Tuesday, December 10, 2002

Check your instructor's name and time:

Myung	8:00		Bode	11:00	
Gasper	9:00		Wahl	12:00	
Liang	9:00		Wahl	1:00	
Bode	10:00		Song	1:00	

Prob.	Possible points	Score
1	20	
2	8	
3	12	
4	10	
5	20	
6	12	
7	12	
8	12	
9	15	
10	25	
11	10	
12	12	
13	20	
14	12	
TOTAL	200	

Instructions:
Show *all* your work on these sheets. Feel free to use the opposite side. Make sure that your final answer is clearly indicated. No calculators, books, notes, etc. are allowed. Good luck!

1. (20 pts.) Compute the derivative of each of the following functions. (Do not simplify your answers.)

(a) $f(x) = x^3 + \dfrac{2}{x} + \arctan x$

(b) $f(x) = \sqrt{x}\, e^{2x}$

(c) $g(t) = \dfrac{t^2 + 2}{\cos(3t)}$

(d) $h(x) = \ln(\tan(2x - 1))$

2. (8 pts.) Use Logarithmic Differentiation to compute the derivative of the following function. (Do not simplify your answer.)

$$f(x) = \frac{e^{x^2}\sqrt{x^3+4}}{(x^2+x+1)^3}$$

3. (12 pts.) Find an equation of the tangent line to the curve

$$x^2y - 4y^3 = 3$$

at the point $(1, -1)$.

4. (10 pts.) Suppose f is a differentiable function defined for all real numbers. Assume that $f'(c) = 0$ at some point c. Is it true that f must have a local maximum or a local minimum at c? Explain your answer.

5. (20 pts.) Consider the function $f(x) = xe^{-x}$. Find:

(a) $\lim_{x\to\infty} f(x)$

(b) $\lim_{x\to-\infty} f(x)$

(c) The local maximum points and the local minimum points of the graph of $f(x)$.

(d) The intervals where the function is concave upward, those where it is concave downward and the inflection points.

(e) Use all this information to sketch the graph of f.

6. (12 pts.) Find the linear approximation of $f(x) = \sqrt[5]{x}$ at $a = 1$ and use it to estimate the value of $\sqrt[5]{0.95}$.

7. (12 pts.) Find the global maximum and global minimum values of $f(x) = x - \ln x^2$ where $1 \leq x \leq e$. Hint: $\ln 2 \approx 0.7, e \approx 2.7$.

8. (12 pts.) Find the most general antiderivative of the function.

(a) $f(x) = 3x^7 - \dfrac{1}{x} + e^2$

(b) $g(t) = \dfrac{2\sqrt[3]{t} + \sqrt{t}}{\sqrt{t}}$

9. (15 pts.) A custard pie is thrown vertically up from the ground with an initial velocity of 48 ft/s. Find the greatest height that it attains. Hint: The downward acceleration due to gravity on earth is 32 ft/s^2.

10. (25 pts.) Find the following limits. Use L'Hospital's Rule where appropriate.

(a) $\lim_{x \to 1} \dfrac{x^{100} - 1}{x^{50} - 1}$

(b) $\lim_{x \to 0} \dfrac{3x}{\sin(7x)}$

(c) $\lim_{x \to \infty} \dfrac{3x^3 + 5x}{2x^2 + 10}$

(d) $\lim_{x \to 0^+} (\sin x)^{x^2}$

11. (10 pts.) Find all asymptotes of the function $f(x) = \dfrac{2x-4}{x^2-4}$. For vertical asymptotes $x = c$, find both limits $\lim\limits_{x \to c^-} f(x)$ and $\lim\limits_{x \to c^+} f(x)$.

12. (12 pts.) A ladder 5 ft long is leaning against a vertical wall. If the base of the ladder is being pushed towards the wall at the rate of 4 inches per second, how fast will the top of the ladder move up the wall when the upper end of the ladder is 4 ft from the ground? Hint: 1 foot = 12 inches .

13. (20 pts.) Find the x-coordinate of the point on the curve $y = \sqrt{4x + \dfrac{1}{x}}$, $x > 0$, which is closest to the point $(2, 0)$. (Justify your answer.)

14. (12 pts.) Let $f(x) = x^3 - 3x + 4$.

(a) Explain why Newton's Method fails when applied to the equation $f(x) = 0$ with initial/first approximations $x_1 = 1$ and $x_1 = -1$.

(b) Use Newton's Method with $x_1 = 2$ to find the second approximation x_2.