



Northwestern University

Name: _____

Id #: _____

Math 214-1 Common Final

Fall Quarter 2000

Thursday, December 7, 2000

Check your instructor's name and time:

Lambros 8:00		Kosygin 11:00	
Gasper 9:00		Gutman 12:00	
Sally 9:00		Pinsky 1:00	
Sally 10:00		Bode 1:00	

Name: _____

Id #: _____

Prob.	Possible points	Score
1	20	
2	20	
3	20	
4	15	
5	20	
6	20	
7	20	
8	15	
9	30	
10	20	
TOTAL	200	

Instructions:

Show *all* your work on these sheets. Feel free to use the opposite side. Make sure that your final answer is clearly indicated. No calculators, books, notes, etc. are allowed. Good luck!

1. (20 pts.) Compute the derivative of each of the following functions. (Do not simplify your answers.)

(a) $f(x) = 9x^7 - 3x^{-5} + \frac{5}{\sqrt{x}}$

(b) $g(t) = (t^3 - 4)^5(t^{5/2} - t^{-3/2})$

(c) $h(t) = \frac{\tan(t - 2)}{3t}$

2. (20 pts.) Find an equation of the tangent line to the curve

$$2x^3 + 2y^3 = 9xy$$

at the point (2,1).

3. (20 pts.) Find the following integrals.

(a) $\int (3 \cos(\pi t) + \cos(3\pi t)) dt$

(b) $\int (7\sqrt{x} + \frac{2}{\sqrt{x}} - 1) dx$

(c) $\int_0^\pi \sin x \cos^2 x dx$

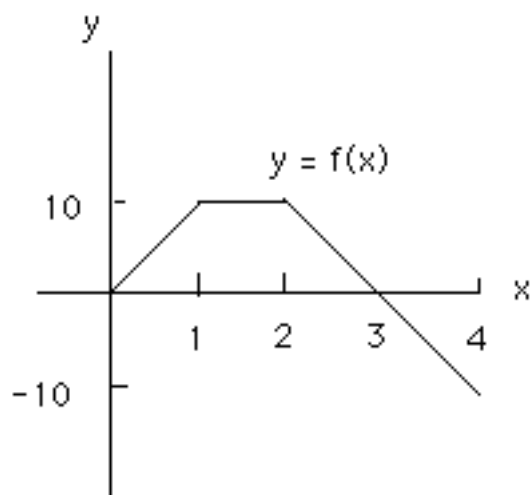
(d) $\int_0^{\sqrt{\pi}} t \sin(t^2) dt$

4. (15 pts.) Suppose $f(x) = x^3 - 3x^2 - 45x$. Find the maximum and minimum values the function attains on the closed interval $[0, 10]$.

5. (20 pts.) Find the dimensions of a rectangular box with square base and no top which has the largest volume provided that its surface area is 192 square inches. (Justify your answer.)

6. (20 pts.) Two parallel sides of a rectangle are being lengthened at the rate of 2 in/sec while the other two sides are shortened in such a way that the figure remains a rectangle with constant area $A = 40 \text{ in}^2$. What is the rate of change of the perimeter P when the length of the increasing sides is 4 in?

7. (20 pts.) Let F and f be functions such that $F'(x) = f(x)$. The graph of $f(x)$ is given below.



- (a) Find where F is increasing and where F is decreasing.

- (b) Evaluate $\int_0^3 f(x) dx$.

- (c) Evaluate $\int_0^4 f(x) dx$.

8. (15 pts.) Use linear approximation to estimate $\sqrt{173}$, using the fact that $\sqrt{169} = 13$.

9. (30 pts.) Suppose $f(x) = \frac{x^2 - 5x + 4}{(x - 3)^2}$.

Here are the first two derivatives of f :

$$f'(x) = \frac{7 - x}{(x - 3)^3} \qquad f''(x) = \frac{2x - 18}{(x - 3)^4}$$

Find:

- a) The vertical and horizontal asymptotes.

- b) The local maximum points and the local minimum points of the graph of $f(x)$.

- c) The intervals where the function is concave upward, those where it is concave downward and the inflection points.

10. (20 pts.) Find the area of the region bounded by the curves $y^2 = x + 1$ and $x = 8$.