Northwestern University

Name: _____

Id #:

Math 214-1 Common Final

Fall Quarter 2001

Monday, December 10, 2001

Check your instructor's name and time:

Chen	8:00		Wahl	9:00	
Loebus	9:00		Bode	10:00	
Bode	11:00		Franks	12:00	
Buchner	1:00		Loebus	1:00	

Prob.	Possible points	Score
1	20	
2	12	
3	12	
4	20	
5	12	
6	20	
7	15	
8	16	
9	18	
10	20	
11	15	
12	20	
TOTAL	200	

Instructions:
Show *all* your work on these sheets. Feel free to use the opposite side. Make sure that your final answer is clearly indicated. No calculators, books, notes, etc. are allowed. Good luck!

1. (20 pts.) Compute the derivative of each of the following functions.(Do not simplify your answers.)

(a) $f(x) = 4x^5 - 2x^2 + 3\sqrt{x} + 1$

(b) $g(t) = (2t^3 - 1)^7$

(c) $h(t) = \dfrac{\cos(2t)}{t^2}$

(d) $f(x) = \sqrt[3]{\sin(x^2 + 1)}$

(e) $f(x) = \displaystyle\int_1^{x^2} \frac{dt}{t}$

2. (12 pts.) Find an equation of the tangent line to the curve

$$x^3 - y + y^2 = 3$$

at the point (1,2).

3. (12 pts.) Use the definition of the derivative to find $f'(x)$ when $f(x) = x^2 + x$.

4. (20 pts.) Consider the function $f(x) = 3x^5 - 5x^3$.

Find:

(a) The local maximum points and the local minimum points of the graph of $f(x)$.

(b) The intervals where the function is concave upward, those where it is concave downward and the inflection points.

(c) Use all this information to sketch the graph of f.

5. (12 pts.)

Use linear approximation to estimate the value of $\sqrt{24}$.

6. (20 pts.) Find the following integrals.

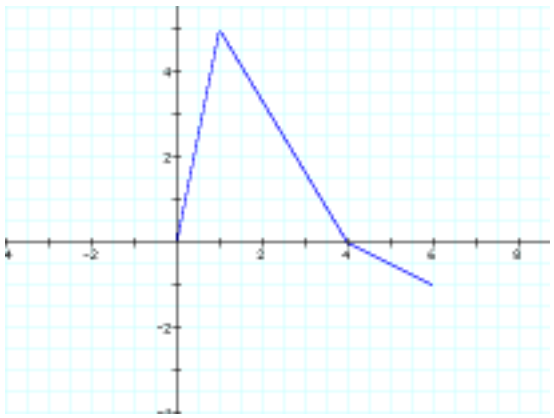
(a) $\int (3\,\sin(t) + \sin(3t))dt$

(b) $\int (2\sqrt{x} + 4x^2 - 2)\,dx$

(c) $\int 5x^2\ \sin x^3\ dx$

(d) $\int_1^4 \dfrac{1}{2\sqrt{x}\ \sqrt{1+\sqrt{x}}}\,dx$

7. (15 pts.) The graph of $f(x)$ is given below.



Evaluate:

(a) $\int_0^4 f(x)dx$

(b) $\int_4^5 f(x)dx$

(c) $\int_0^5 f(x)dx$

(d) $\int_5^4 f(x)dx$

(e) If $F(x) = \int_x^0 f(t)dt$, what is $F'(5)$?

8. (15 pts.) An astronaut is standing on a platform 3 meters above the moon's surface and throws a rock directly upward with an initial velocity of 16 m/s. Given the acceleration due to gravity on the moon's surface is -1.6 m/s^2, how high will the rock travel?

9. (18 pts.) Find the following limits:

(a) Let $f(x) = \dfrac{x^2 - 1}{3x^2 - 3x - 6}$

(i) $\lim_{x \to -1^-} f(x)$

(ii) $\lim_{x \to -1^+} f(x)$

(iii) $\lim_{x \to 2^-} f(x)$

(iv) $\lim_{x \to 2^+} f(x)$

(v) $\lim_{x \to \infty} f(x)$

(vi) $\lim_{x \to -\infty} f(x)$

(vii) Find all vertical and horizontal asymptotes of the function f.

(b) $\lim_{x \to 0} \dfrac{3x}{\sin(7x)}$

10. (20 pts.) First sketch the region bounded by the given curves $y = x^2 - 1$ and $y = x + 1$ and then find the area of that region.

11. (15 pts.) A ladder 10ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 ft from the wall?

12. (20 pts.) Suppose you have to make a cylindrical can of volume $25\pi ft^3$. The top and the bottom circles are to be made of cardboard which costs $\$0.5/ft^2$ and the side is made by bending a rectangular piece of metal sheet which costs $\$5/ft^2$. What should be the dimensions of the cylinder if you want to minimize the cost? (Justify why your answer is a minimum.)