Northwestern University

Name: ______________________

Id #:

Math 214-2 Final Exam

Winter Quarter 2003

Wednesday, March 19, 2003

Check your instructor's name and section:

Myung	8:00		Liang	12:00	
Myung	9:00		Song	12:00	
Liang	10:00		Song	1:00	
Bode	11:00				

Prob.	Possible points	Score
1	18	
2	12	
3	10	
4	12	
5	8	
6	8	
7	12	
8	10	
9	12	
10	12	
11	8	
12	12	
13	12	
14	10	
15	10	
16	12	
17	10	
18	12	
TOTAL	200	

Instructions:
Show *all* your work on these sheets. Feel free to use the opposite side. Make sure that your final answer is clearly indicated. No calculators, books, notes, etc. are allowed. Good luck!

1. (18 points) Evaluate the following integrals.

(a) $\int (\sqrt{t} - \frac{1}{t} + \frac{1}{t^2+1})\ dt$

(b) $\int \sin^2 t \cos^3 t\ dt$

(c) $\int_0^1 (2x+1)e^x\ dx$

2. (12 points) Evaluate:

$$\int \frac{2\ dx}{x^3 - x}$$

3. (10 points) Evaluate:

$$\int \cos(\ln t)\ dt$$

4. (12 points) Use the trigonometric substitution $x = 2\sin\theta$ to evaluate the integral.

$$\int \sqrt{4 - x^2} \; dx$$

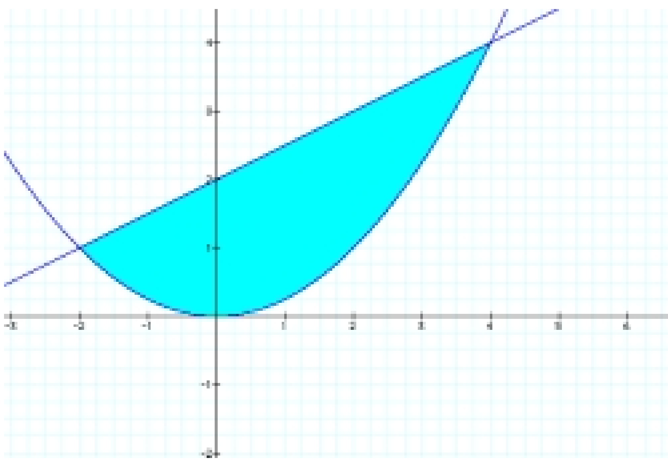
5. (8 points) Use the Comparison Theorem to determine whether $\int_4^\infty \frac{dx}{\sqrt{x}-1}$ is convergent or divergent.

6. (8 points) Last week the math department hired a typist to help type final calculus exams. His instantaneous speed (measured in thousands of characters per hour) was measured each hour and the results are given below:

Time	8am	9	10	11	12
Speed	6	4	1	3	5

Use Simpson's Rule to approximate the total number of characters typed between 8am and noon.

7. (12 points) Let R be the region bounded by the curves $x = 2y - 4$ and $y = \dfrac{x^2}{4}$.



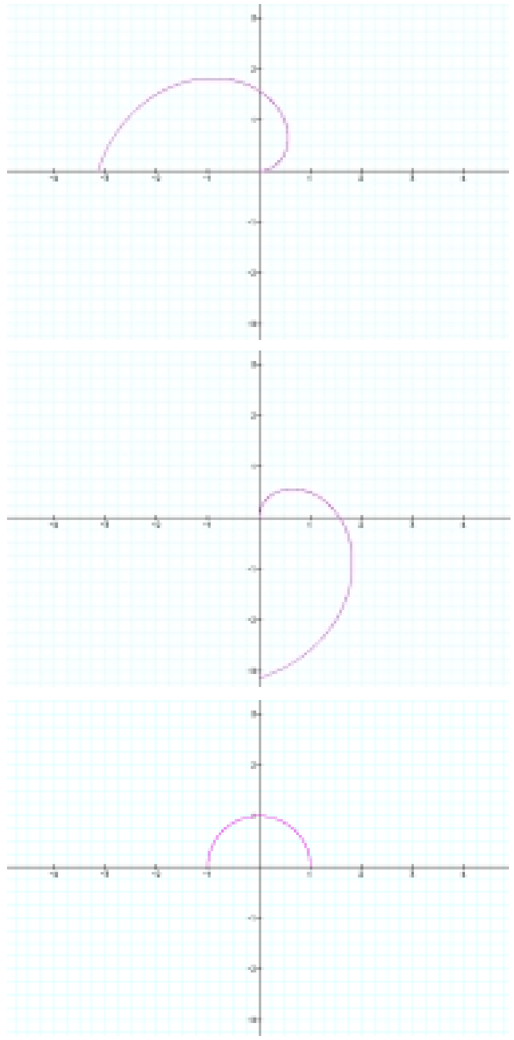
Set up and do not evaluate the integral for the volume of the solid obtained by rotating the region R about

(a) the x -axis,

(b) the line $x = 5$.

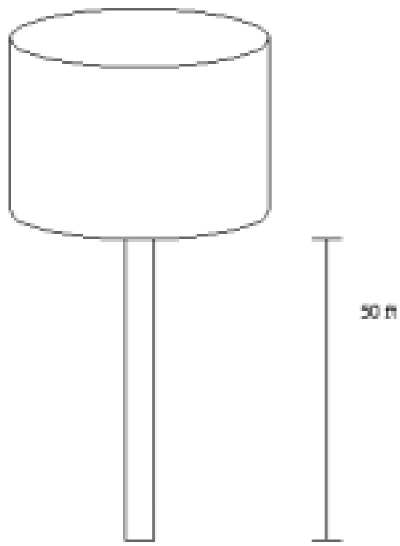
8. (10 points) Consider the parametric curve $x(t) = t\cos t, y(t) = t\sin t, 0 \le t \le \pi$.

(a) Circle the curve that represents this parametric equation.



(b) Set up an integral to compute the length of this curve.

9. (12 points) A cylindrical water tank whose radius is 10 ft and height is 20 ft is on top of a tower 50 ft high. The density of water is approximately 60 pounds per cubic foot. Set up an integral for the work required to fill this tank by pumping water up from the ground level.



10. (12 points) Suppose that X measures the time (in hours) it takes for students to complete the Math 214-2 final exam. Assume that all students are done within two hours and the density function for X is given by

$$p(x) = \begin{cases} \frac{x^3}{4} & \text{if } 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) What proportion of students take between 1 and 2 hours to finish the exam?

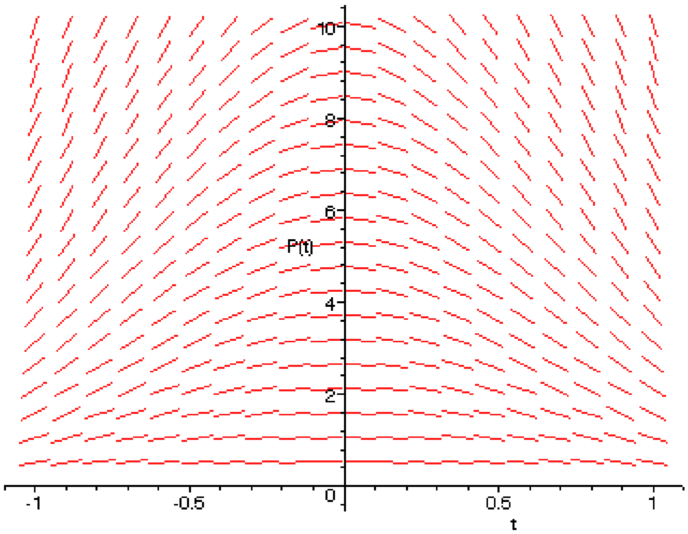
(b) What is the mean time for students to finish the exam?

11. (8 points) Suppose we know that the amount of a certain substance S grows at a rate proportional to the square of its difference from some maximal amount M. Set up a differential equation which models this situation. You do not need to solve this equation!

12. (12 points) For this problem consider the initial value problem:

$$\frac{dy}{dx} = -2xy \qquad y(0) = 5$$

(a) Below is the slope field for the differential equation $\frac{dy}{dx} = -2xy$. Sketch a solution to the initial value problem.



(b) Solve the initial value problem.

13. (12 points) Test the following series for convergence or divergence.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{3n^2}{n^3 - 5}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(\ln n)^2}$

14. (10 points) Does the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ converge? Does it converge absolutely?

15. (10 points) Find the sum of the following convergent series.

$$\sum_{k=1}^{\infty} \frac{(-2)^{k-1}}{3^{k+1}}$$

16. (12 points) Find the interval of convergence including endpoints if any for the following power series.

$$\sum_{n=1}^{\infty} \frac{(x-3)^n}{n2^n}$$

17. (10 points) Find the Taylor polynomial of degree 3 for the function $f(x) = \dfrac{1}{x}$ at $a = 1$.

18. (12 points) (a) Use the Maclaurin series for $\cos x$ to find the Maclaurin series for the function $f(x) = \cos(x^3)$.

(b) Use the first three non-zero terms of the series you found in part (a) to approximate the definite integral.

$$\int_0^1 \cos(x^3)\ dx$$