Math 214-2, Review problems for Test # I

1. Evaluate the integrals.

(a) $\displaystyle\int_1^4 (\sqrt{x} - \frac{1}{\sqrt{x}} + \frac{1}{x})dx$ $\qquad$ (b) $\displaystyle\int \frac{e^{\sqrt{p+2}}}{\sqrt{p+2}}\ dp$

(c) $\displaystyle\int \frac{3x+2}{\sqrt{1-x^2}}\ dx$ $\qquad$ (d) $\displaystyle\int \frac{\cos(\sqrt{t})}{\sqrt{t}}\ dt$

(e) $\displaystyle\int \arcsin x\ dx$ $\qquad$ (f) $\displaystyle\int \frac{x+1}{x^2(x-1)}\ dx$

(g) $\displaystyle\int \ln(1+x^2)\ dx$ $\qquad$ (h) $\displaystyle\int \frac{dx}{\sqrt{4-x^2}}$

(i) $\displaystyle\int e^{2t}\ \cos 2t\ dt$ $\qquad$ (j) $\displaystyle\int x^3 \ln x\ dx$

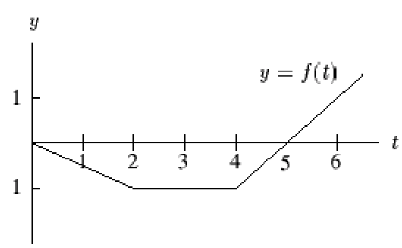
(k) $\displaystyle\int (p^3+6p)\ \sin p\ dp$ $\qquad$ (l) $\displaystyle\int 1+\tan^2\theta\ d\theta$

(m) $\displaystyle\int_0^1 x\sqrt{9-x^2}\ dx$ $\qquad$ (n) $\displaystyle\int \sin^3 x \cos^2 x\ dx$

(o) $\displaystyle\int \frac{\cos^4 x}{1-\sin^2 x}\ dx$ $\qquad$ (p) $\displaystyle\int \frac{x+9}{x^3+9x}\ dx$

(q) $\displaystyle\int_0^1 \theta^3 e^{\theta^2} d\theta$ $\qquad$ (r) $\displaystyle\int \frac{1}{x^2\sqrt{x^2+4}}\ dx$

2. The graph of f is shown below. Evaluate $\displaystyle\int_0^5 f(t)\ dt$, $\displaystyle\int_5^6 f(t)\ dt$, $\displaystyle\int_6^0 f(t)\ dt$ and $\displaystyle\int_0^6 |f(t)|\ dt$.



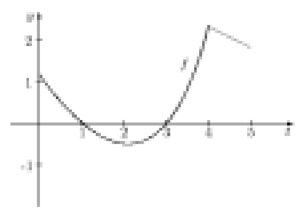
3. Evaluate: $\displaystyle\frac{d}{dx}\int_{x^2}^{\pi} \frac{\sin t}{t}\ dt$

4. Suppose water is flowing into a tank at a rate of $r(t)$ gallons per minute, where t is in minutes.
(a) Write a definite integral expressing the total amount of water which flows into the tank in the first hour.
(b) A table of values of $r(t)$ is given below. Suppose $r(t)$ is steadily increasing. Use the table to find lower and upper estimates for the amount in part (a).

t(min)	0	10	20	30	40	50	60
$r(t)$(gallons/min)	5	6.2	7	7.6	8.2	8.7	9

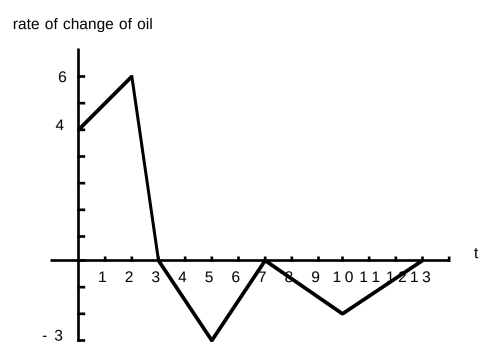
(c) Use Simpson's Rule to approximate the amount of water in part (a).

5. Let $g(x) = \int_0^x f(t)\,dt$, where f is the function whose graph is shown below.



(a) At what values of x does g have local maximum and local minimum values?
(b) On what intervals is g increasing/decreasing?
(c) On what intervals is g concave upward/downward?
(d) Where does g attains its absolute maximum values?
(e) Sketch the graph of g.

6. The graph below gives the (net) rate of change of the amount of oil in the water from a tanker spill. Thus at $t = 2$ days the oil is flowing into the water at the rate of 6 million barrels per day, while at $t = 5$ days the oil is being removed from the water at a rate of 3 million barrels per day.



(a) If at $t = 0$ there is 10 million barrels of oil in the water, find the amount of oil in the water after $t = 2$, 3, 5 days.
(b) When is there the most oil in the water?
(c) When is the graph of oil vs. time increasing? decreasing? concave up ? concave down?