



Math 214-2 Common Final

Winter Quarter 2002

Tuesday, March 19, 2001

Check your instructor's name and section:

Wahl	8:00		Bode	11:00	
Wahl	9:00		Buchner	1:00	
Hornbostel	10:00		Hornbostel	1:00	

Prob.	Possible points	Score
1	8	
2	8	
3a,b	16	
3c,d	18	
3e,f	18	
4	8	
5a	12	
5b	12	
6	16	
7	16	
8	16	
9	12	
10	12	
11	10	
12	18	
TOTAL	200	

Instructions:

Show *all* your work on these sheets. Feel free to use the opposite side. Make sure that your final answer is clearly indicated. No calculators, books, notes, etc. are allowed. Good luck!

1. (8 points) The velocity $v(t)$ (in feet per second) of a bicycle is shown in the following table at one second intervals. Use Simpson's rule to approximate the total distance traveled from $t = 0$ to time $t = 4$.

t	0	1	2	3	4
v(t)	3	4	5	4	7

2. (8 points) Use logarithmic differentiation to find $\frac{dy}{dx}$ for $y = x^x$.

3. (52 points) Evaluate the following integrals:

(a)(8 points) $\int \sec^2 x \tan x \, dx$

(b)(8 points) $\int_0^2 \frac{dx}{x^2 + 4}$

(c)(8 points) $\int \arctan x \, dx$

(d)(10 points) $\int e^x \cos x \, dx$

(e)(8 points) $\int (x^2 + 1) \cos x \, dx$

(f)(10 points) $\int \frac{2x + 6}{x^2 + 4x + 5} \, dx$

4. (8 points) Evaluate the following integral if it converges.

$$\int_1^{\infty} e^{-x} dx$$

5. (24 points)

(a) (12 points) Use a trigonometric substitution to evaluate the integral:

$$\int \frac{x^3 dx}{\sqrt{9-x^2}}$$

(b) (12 points) Use partial fractions to evaluate the integral:

$$\int \frac{5x^2 + 9x + 3}{x^3 + 2x^2 + x} dx$$

6. (16 points) Find the following limits: (8 points each)

(a) $\lim_{x \rightarrow 0} \frac{6x - x^3 - 6 \sin x}{x^5}$

(b) $\lim_{x \rightarrow 0} (1 + 2x)^{1/x}$

7. (16 points) Consider the plane region R enclosed by the parabola $y = 4 - x^2$ and the x -axis. **Set up and do not evaluate** the integral for the volume of the solid obtained by revolving R about

(8 points each)

(a) the x -axis

(b) the line $x = 3$

8. (16 points)

(a)(8 points) Set up the integral for the arc length of the curve $y = 1 - x^2$ between $x = 0$ and $x = 100$. Do not evaluate the integral!

(b)(8 points) Rotate $y = x^2, 0 \leq x \leq 1$ around the y -axis and set up the integral for the area of the surface of revolution. Do not evaluate the integral!

9. (12 points) Solve the initial value problem $\frac{dy}{dx} = \frac{1}{2}y^{-3}$, $y(0) = 1$ for y .

10. (12 points) A fish tank has a rectangular base 2 ft long and 1 ft wide. The sides are 1 ft high, and the tank is full of water. How much work will it take to empty the tank by pumping the water out over the top of the tank? For simplicity, take the density of water to be 60 lb/ft^3 .

11. (10 points) Find the Taylor polynomial of degree 4 for the function $f(x) = \sin x$ at $a = \frac{\pi}{2}$.

12. (18 points) As you know, when a course ends, students start to forget the material they have learned. Let $y = f(t)$ be the fraction of the original material remembered t weeks after the course has ended. One model (called the NU Calculus model) assumes that the rate of change of y is proportional to y . Suppose that in a certain calculus course a student worked really hard and knew 100 % of the material at the end of the course. Assume further that four weeks after the course ends the student remembers 50 % of the material. How long was the student able to remember at least 80 % of the material? Note that $\ln 0.8 \approx -0.2$ and $\ln 0.5 \approx -0.7$.