Math 214-3, Final Examination

1. Find the position vector of a particle that has acceleration $\mathbf{a}(t) =< 1,\ 2,\ 2t >$ and initial velocity $\mathbf{v}(0) =< 1,\ -1,\ 0 >$ and initial position $\mathbf{r}(0) =< 1,\ 0,\ 1 >$.

2. Use differential to estimate $\sqrt{2 \cdot (2.03)^3 + 2.97^2}$.

3. Find the arc length of the curve $\mathbf{r}(t) = (2e^t,\ e^{-t},\ 2t)$ for $0 \leq t \leq 1$; Find the curvature of the curve at $t = 1$.

4. Find the directional derivative of the function $f(x, y, z) = x^2y + xyz + e^z y$ at the point (1, -2, 0) in the direction of $\mathbf{v} =< 1,\ 2,\ -2 >$; A particle moves along the path $\mathbf{r}(t) = (t^2,\ -2t,\ e^t - t^2)$, find $df(\mathbf{r}(t))/dt$ at $t = 1$.

5. Find the area of triangle with vertices $(1,\ -1,\ 0)$, $(0,\ 2,\ 1)$, $(-1,\ 0,\ 1)$ and find the equation of the plane containing the triangle.

6. $z = f(x, y)$ is a function that satisfies the equation $x^2 + 2y^2 + 4z^2 = 10$. Find $\partial z/\partial x$, $\partial z/\partial y$ and $\partial^2 z/\partial x^2$; Find the tangent plane of the ellipsoid $x^2 + 2y^2 + 4z^2 = 10$ at the point $(2, 1, -1)$.

7. Find all the intersections of the two curves $\mathbf{r}(t) = (t,\ t^2,\ t^3)$ and $\mathbf{r}(t) = (t-1,\ e^{(t-2)},\ t-1)$ (yes, they do intersect!); Find the angle of intersection at the intersection point.

8. Find the absolute maximum and absolute minimum values of $f(x, y) = xy$ on the region bounded by $y = 3 - x^2$ and $y = -1$.

9. Locate and classify all the critical points of the function $f(x, y) = 3x - x^3 - 3xy^2$.

10. Find the point closest to the origin on the curve of intersection of the plane $2y + 4z = 5$ and the cone $z^2 = 4x^2 + 4y^2$.