Math 214-3 section 51

Name: ______________________

Final Examination

I.D. # ______________________

Fall 2000

Instructions: Write your name and I.D. number above. Calculators are allowed, but you must show all work on these pages, and make sure that your final answer is clearly shown. **No books or tables** are allowed.

1. Consider a triangle with vertices $P(1,4,0), Q(1,0,-2)$,and $R(3,2,0)$.
   (a) (6 points) Compute the cosine of the angle in the triangle at the point R.
   (b) (6 points) Compute the area of the triangle PQR.
   (c) (6 points) Find the parametric equation of a line through Q and R .
   (d) (6 points) Find the parametric equation of a line through P which is perpendicular to the plane containing the triangle PQR.
   (d) (6 points) Find the parametric equation of a line through P which is in the plane containing the triangle PQR and perpendicular to the line segment PR.

2. (20 pts) A parametric curve in the plane is given by $x(t) = \sin t,\ y(t) = \ln(t+1)$. Find dy/dx and d^2y/dx^2 at the point $(0,0)$.

3. (25 pts) If $\vec{v}(t) = 2\vec{i} + (2t^2 - 2t)\vec{j} + 8t^3\vec{k}$, represents the velocity of a particle in space at time t, find the following
   (a) the postion vector $\vec{r}(t)$, if $\vec{r}(1) = (1,0,0)$.
   (b) the acceleration vector $\vec{a}(t)$
   (c) $\int_0^3 \vec{v}(t)dt$
   (d) The value of t when the particle is moving in a direction parallel to the line

$$x - 2 = \frac{y}{2} = \frac{z-2}{32}.$$

4. (15 pts) Two lines in space intersect at the point $P(1,2,1)$. One of the lines is parallel to

$$\frac{x-2}{2} = \frac{y}{3} = \frac{z+2}{2}$$

and the other is parallel to

$$x + 2 = \frac{y-1}{4} = \frac{4-z}{3}.$$

Find the equation of the plane containing these two lines.

5. The function $f(x,y,z)$ is defined by $f(x,y,z) = x^2 + 2xy + yz^3$.
   (a) (10 pts) Find the equation of the tangent plane to the surface $f(x,y,z) = 19$ at $P(2,3,1)$.
   (b) (10 pts) Find a direction $\vec{u}$ such that $D_{\vec{u}}f(2,3,1) = 0$.

6. (15 points) Find the length of the parametric curve $\vec{f}(t) = \langle t^2/\sqrt{2}, t^3/3, t \rangle$ for $0 \le t \le 4$

7. (20 points) Three positive numbers x, y, z satisfy $x+2y+3z = 60$. What is the maximum possible value of their product?

8. (20 points) Use Lagrange multipliers to find the maximum of $f(x, y, z) = x - y + z$ subject to the constraint $x^2 + 4y^2 + 9z^2 = 49$.

9. (15 points) If $z = x^2y^2 + xy$ and $x = s - t$ and $y = t + e^s$, find $\dfrac{\partial z}{\partial t}$ and $\dfrac{\partial z}{\partial s}$.

10. A child riding a merry-go-round is holding a ball so that the position of the ball after t seconds is given by

$$\vec{R}(t) = \left\langle \frac{32\cos(\pi t)}{\pi}, \frac{32\sin(\pi t)}{\pi}, 9 \right\rangle,$$

using feet as units. (x and y axes are horizontal and z axis is vertical.) At time $t = 1$ the child releases the ball and it flies off the merry-go-round and hits the ground.

(a) (5 points) What are the position and velocity (vector) of the ball at the instant the child releases it?

(b) (10 points) How long after the ball is released does it hit the ground? The acceleration due to gravity is $-32 ft/sec^2$.

(c) (5 points) What are the x and y co-ordinates of the point where the ball hits the ground? (The z co-ordinate of this point is 0).