Northwestern University

Name: ______________________

Id #:

Math 214-3, Final Exam

Spring Quarter 2002

Tuesday, June 11, 2002

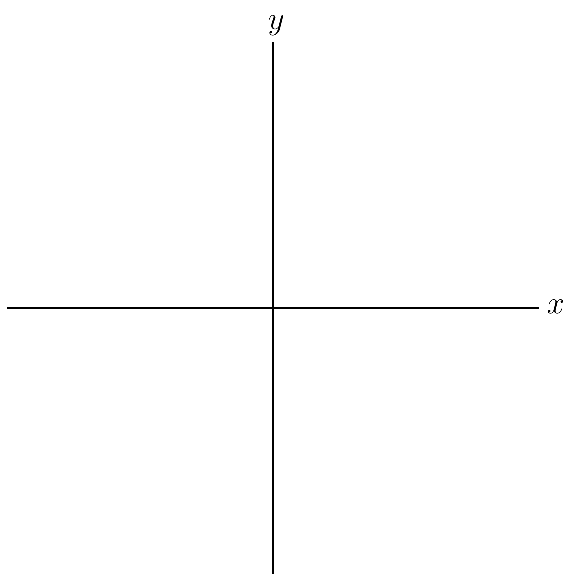
Check your instructor's name and section:

Klimenko	9:00		Klimenko	10:00	
Bode	11:00		Mauger	1:00	

Prob.	Possible points	Score
1	8	
2	12	
3	12	
4	18	
5	12	
6	10	
7	12	
8	10	
9	12	
10	20	
11	20	
12	16	
13	26	
14	12	
TOTAL	200	

Instructions:
Show *all* your work on these sheets. Feel free to use the opposite side. Make sure that your final answer is clearly indicated. No calculators, books, notes, etc. are allowed. Good luck!

1. (8 points) Sketch the graph of $r = 3 + 2\cos\theta$. Does the graph repeat itself as θ ranges from 0 to 2π?



2. (12 points)

   (a) (6 points) Write an equation of the line tangent to the parametric curve $x = \cos^3 t, y = \sin^3 t$ at $t = \frac{\pi}{4}$.

   (b) (6 points) Find the length of the given curve for $0 \leq t \leq \frac{\pi}{4}$.

   Hint: Set up the integral and factor out $\sin^2 t + \cos^2 t$.

3. (12 points) Suppose that a dove is flying so that its acceleration vector at time t is given by $\mathbf{a}(t) =< \sin(t), \cos(t), 1 >$, its initial velocity is $\mathbf{v}(0) =< 0, 1, 1 >$, and its initial position is $\mathbf{r}(0) =< 0, 0, 0 >$.

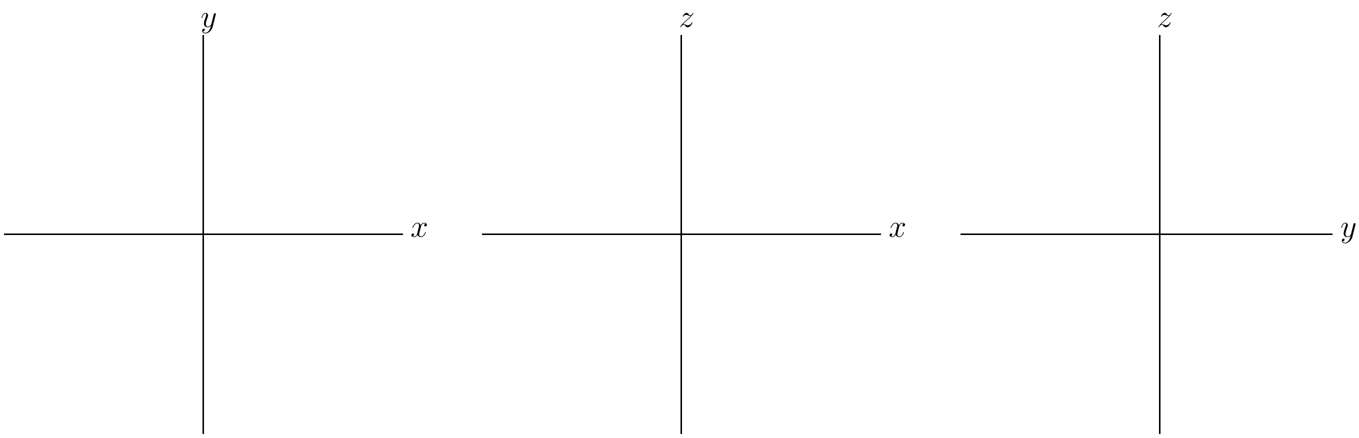
   (a) (6 points) Find the velocity vector $\mathbf{v}(t)$ of the dove at time t.

   (b) (2 points) Find the speed of the dove when $t = \pi$, don't simplify.

   (c) (4 points) Find the position vector $\mathbf{r}(t)$ of the dove at time t.

4. (18 points) Let S be the surface $z^2 - x^2 - y^2 = 0$.

(a) (6 points) Sketch and label the traces of S in the xy-, xz- and yz-plane.

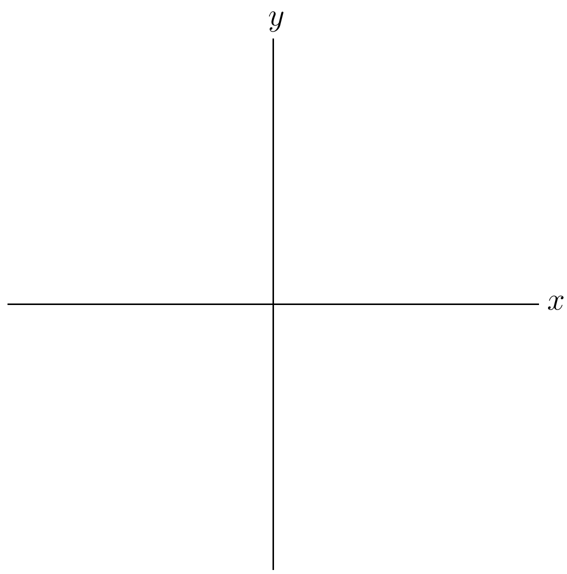


(b) (4 points) Sketch the surface S.

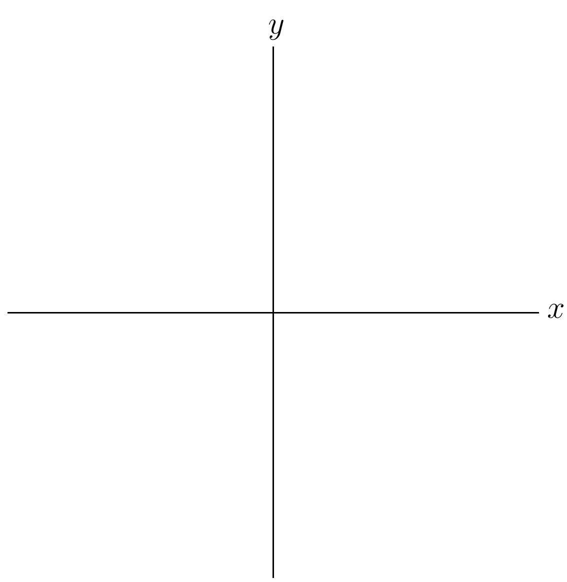
(c) (8 points) Find the equation of the tangent plane to S at $(3, 4, 5)$.

5. (12 points) Let $f(x, y) = \sqrt{x^2 + y^2 - 4}$

(a) Find the largest domain of definition for the function f and sketch the domain.



(b) Sketch and label the level curves of f which go through the points $(2, 2), (3, 2)$ and $(2, 4)$. Indicate on your sketch (with an arrow) the direction of the gradient vector for this function at the point $(2, 2)$.



6. (10 points) Find $\dfrac{\partial z}{\partial x}$ as a function of x, y and z assuming that $z = f(x, y)$ satisfies the equation $x^3 + y^3 + z^3 = xyz$.

7. (12 points) Write an equation of the plane that passes through $A(2, 1, -1)$ and is perpendicular to both the planes $x - y + 5z = 0$ and $2x + y = 3$.

8. (10 points) Find the distance between the point $P(1, 0, 1)$ and the plane $x + 2y - z = 4$. Hint: Find a point on the plane, and use the dot product.

9. (12 points) Let $z = x^2 + y^2$, and let x and y be functions of s and t. Suppose $x = 1$ and $y = 2$ when $s = 0$ and $t = 0$, and $\left.\dfrac{\partial x}{\partial s}\right|_{(0,0)} = 3$ and $\left.\dfrac{\partial y}{\partial s}\right|_{(0,0)} = 4$.

Find $\left.\dfrac{\partial z}{\partial s}\right|_{(0,0)}$.

10. (20 points) Given the pyramid with vertices $A(2, 1, -1), B(3, 0, 2), C(5, 1, 1)$ and $D(0, -1, 3)$, find

(a) (8 points) the area of the base triangle ΔABC,

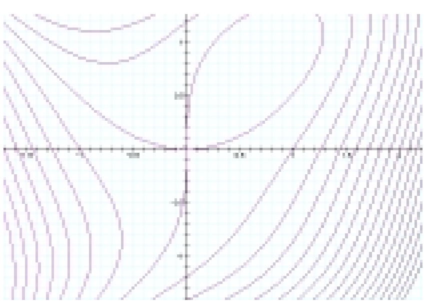
(b) (6 points) the volume of the pyramid (the volume of a pyramid is 1/6 the volume of the corresponding parallelepiped),

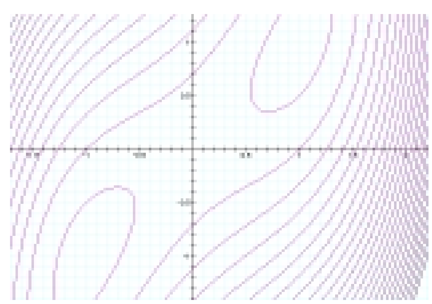
(c) (6 points) the height, i.e. altitude, of the pyramid from D to the base ΔABC.

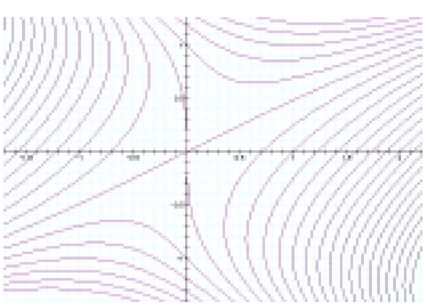
11. (20 points)

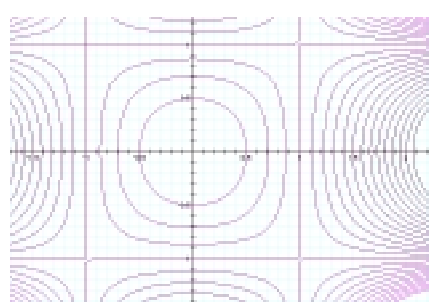
(a) (15 points) Classify all critical points of the function $f(x, y) = -x^4 + 4xy - 2y^2 + 1$.

(b) (5 points) Match the function $f(x, y)$ in part (a) to the plot of its level curves. Explain. Hint: Locate the critical points that you found in part (a).









12. (16 points) Use Lagrange multipliers to find the greatest and smallest values that the function $f(x, y) = xy$ attains on the ellipse $\dfrac{x^2}{8} + \dfrac{y^2}{2} = 1$.

13. (26 points) Consider the function $f(x, y) = \dfrac{1}{1 + x + y}$.

(a) (4 points) Calculate $\nabla f(3, 6)$.

(b) (6 points) Find the equation(s) of the line in the xy plane through $P = (3, 6)$ along which the function f increases fastest in one direction, and decreases fastest in the opposite direction.

(c) (6 points) Find the directional derivative of f at the point $(3, 6)$ in the direction towards $(9, 14)$.

(d) (10 points) Use differentials to find the approximate value of $f(3.02, 6.05)$.

14. (12 points) **Choose either problem I or II, but not both! Clearly mark the one you want graded.**

I. In 1754, Virginia sent a small force under young George Washington to capture Fort Duquesne, a French post located on the present site of Pittsburgh. General Washington's artillery, which is situated 1000 feet from the fort, fires a cannon at an angle of 60 degrees from the horizontal with a muzzle velocity of 160 ft/sec.

(a) At what time, t, after the cannon was fired, does the cannonball land? (Assume the ground around the battlefield is level)

(b) How far from the cannon will the cannonball land? Will it hit Fort Duquesne?

II. Find the work done by the force field $\mathbf{F}(x, y) = xy\mathbf{i} + x^2\mathbf{j}$ on a particle that moves along the curve $\mathbf{r}(t) = t\mathbf{i} + 2t\mathbf{j}, 0 \leq t \leq 1$.