Math 214-3 Final March 15, 2000 R.C. Robinson

No calculators, no books, no notes.

Start each problem on a new page.

(1) (12 Points) Give the equation of the plane which contains the point $P(2,5,7)$ and the line $\langle x,y,z\rangle = \langle 2+3t, 4+2t, 6+t\rangle$.

(2) Let $\mathbf{r}(t) = 3\cos(t)\mathbf{i} + 3\sin(t)\mathbf{j} + 5t^2\mathbf{k}$ be a curve space.
   (a) (20 Points)Find the velocity, speed, and acceleration for this curve.
   (b) (10 Points) Find the scalar component of the acceleration along the velocity at an arbitrary time t.
   (c) (10 Points) Give the parametric equation of the tangent line at $t = \pi/3$.
   (d) (10 Points) Give the integral which gives the length of the curve from $t = 0$ to $t = 2$.

(3) Let $f(x,y) = 3x^2 - y^2$.
   (a) (18 Points) Draw the level curves $f(x,y) = C$ for $C = -1, 0, 1$ on one sketch, labeling each level curve. One the same sketch, put the $\nabla f_{(1,3)}$.
   (b) (10 Points) Give the equation of the tangent plane of the graph $z = f(x,y)$ at the point $P(1,3)$.

(4) Let $w = f(x,y)$, and assume that the gradient at $P(3,3\sqrt{3})$ is $\langle 4,-2\rangle$, $\nabla f_{(3,3\sqrt{3})} = \langle 4,-2\rangle$.
   (a) (16 Points) Calculate $\dfrac{\partial w}{\partial r}$ and $\dfrac{\partial w}{\partial \theta}$ at this point, where r and θ are the polar coordinates.
   (b) (10 Points) What is the direction of greatest increase at this point.
   (c) (10 Points) What is the directional derivative in the direction of the vector $\mathbf{v} = \langle 4,3\rangle$.

(5) Consider the surface given by the equation
$$yz^2 - x^2yz + xy^2 = 5.$$
   (a) (10 Points) Find the equation of the tangent plane at the point $P(2,-1,3)$.
   (b) (14 Points) Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ at $P(2,-1,3)$.
   (c) (10 Points) Using the linear approximation, approximate the z on the level surface that corresponds with $x = 2.1$ and $y = -1.2$.

(6) Let $f(x,y) = x^3 - 3xy + y^3$.
   (a) (10 Points) Find the critical points.
   (b) (10 Points) Classify each critical point as a maximum, minimum, or neither.

(7) (20 Points) Find the maximum and minimum of $f(x,y) = x^2 - x + y^2 - 2y$ subject to the constraint $x^2 + y^2 = 20$.