Final Exam

MATH 221-0

June 11, 2003

Name:

No books, notes, or calculators permitted. Answers need not be in decimal format, of course: for example, $\sqrt{2}$ is fine, and preferable to $1.414\ldots$ Remember that sometimes the best you can do in solving a differential equation is to write the solution parametrically, as the curve defined by a (non-differential) equation; that's ok.

Note that the problems are worth varying amounts, and in particular the first one is worth quite a lot.

Show your work; not doing so may lose you points. Write on the back of the page if you run out of room.

	1 25 pts	2 12 pts	3 10 pts	4 10 pts	6 10 pts	7 10 pts	**Total** 77 pts
Score							

(1) *(5 pts. each)* Find *all solutions* to each of the following differential equations:

(a) $y'' - 5y' + 6y = 0$

ANSWER: _____

(b) $y' = (1 - 2x)y^2$

ANSWER: _____

(c) $y' - 2y = t^2 e^{2t}$

ANSWER: _____

(d) $(\cos y - te^y)y' + 2t - e^y = 0$

ANSWER: _____

(e) $y'' + 2y' = 4 \sin 2t$

ANSWER: _____

(2) *(12 pts.)*

(a) *(9 pts.)* Find the general solution of

$$(t^2 + 3)y'' - 2y = 0$$

using power series about the point $t_0 = 0$. Find the recurrence relation and the first four nonzero terms of each of the two linearly independent solutions. *(Note: you are not being asked to solve the recurrence relation to find a formula for the general term in the series.)*

ANSWER: _____

(b) *(3 pts.)* Determine a lower bound for the radius of convergence of the answer above.

ANSWER: _____

(3) *(10 pts.)* An object of mass $m = 1\text{kg}$ is attached to a spring with spring constant $k = 9\text{N/m}$ and damping constant $\gamma = 6\text{kg/s}$. The spring is stretched $+1$ meter from its equilibrium position, and released with an initial velocity of -5 meters per second. Find the greatest compression attained by the spring, i.e. the minimum value achieved by the position.

ANSWER: _____

(4) *(10 pts.)* A ten-liter fish tank is initially full of salt water, with a salt concentration of two grams per liter. Water with a salt concentration of one gram per liter is pumped in at a rate of one liter per hour, and mixes with the contents of the tank; the mixture is allowed to overflow the tank to maintain the total volume at ten liters. Find the concentration of salt after 9 hours.

ANSWER: _____

(5) *(10 pts.)* Find the solution to the initial value problem

$$\vec{x}'(t) = \begin{pmatrix} 1 & -2 \\ 5 & -5 \end{pmatrix} \vec{x}(t),$$

with

$$\vec{x}(0) = \begin{pmatrix} 1 \\ 3 \end{pmatrix}.$$

ANSWER: _____

(6) *(10 pts.)* Find the most general solution to the system

$$\vec{x}'(t) = \begin{pmatrix} 0 & -1 \\ 1 & 2 \end{pmatrix} \vec{x}(t).$$

ANSWER: _____

(7) *(10 pts.)* Find all solutions $\vec{x}(t)$ to the equation

$$\vec{x}'(t) = \begin{pmatrix} -2 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 5 \end{pmatrix} \vec{x}(t).$$

for which

$$\lim_{t \to +\infty} \vec{x}(t) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

ANSWER: _____