

1. Compute the following derivatives: (*do not simplify*)

(a)  $\frac{d}{dx} x^3 - 5x + 1$

(b)  $\frac{d}{dx} \frac{x^3 + 1}{x^2 + x - 2}$

(c)  $\frac{d}{dx} \sqrt{x^2 + 1} \sin 7x$

(d)  $\frac{d}{dx} \left(x^{\frac{1}{7}} - 2\right)^{\frac{5}{8}}$

(e)  $\frac{d}{dx} \int_0^{x^2} \sqrt{1+t^4} dt$

2. Compute the following integrals:

(a)  $\int_0^1 3x^2 - 2x + 1 dx$

(b)  $\int_0^\pi \cos^5 2x \sin 2x dx$

(c)  $\int_1^4 \frac{x^3 + 1}{\sqrt{x}} dx$

(d)  $\int_2^3 x\sqrt{3+2x} dx$

3. Give the definition of the derivative of a function  $y = f(x)$ ; use this definition to compute the derivative of  $y = \frac{1}{x+1}$ .

4. Given  $x^2 + y^2 = 8$ , use implicit differentiation to compute  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$ . Give the answers in terms of  $x$  and  $y$ . Find the equation of the tangent and normal lines to the graph of  $x^2 + y^2 = 8$  which pass through the point  $(2, 2)$ .

5. A girl is flying a kite, which is at a height of 120 feet. The wind is carrying the kite horizontally away from the girl at a speed of 10 feet per second. How fast must the kite string be let out when the string is 150 feet long?

6. Find the area of the bounded region between the curves  $x + y = 1$  and  $y = 7 - x^2$ .

7. If  $f(x) = \frac{2x^2}{x^2 - 9}$ , then  $f'(x) = \frac{-36x}{(x^2 - 9)^2}$  and  $f''(x) = \frac{108(x^2 + 3)}{(x^2 - 9)^3}$ .

(a) Sketch the graph of  $f(x)$ .

Determine and indicate on the graph:

(b) the asymptotes,

(c) where the function is increasing and where it is decreasing,

(d) where the function is concave upwards and where it is concave downwards,

(e) the inflection points,

(f) the local maxima and the local minima,

(g) the critical points.

8. Find the dimensions of the rectangle of maximum area inscribed inside an isosceles triangle with height 4 cm and base 6 cm. Assume that one side of the rectangle lies along the base of the triangle.

9. A ball is thrown upward with an initial velocity of 48 ft/s from the top of a building 160 ft tall, then falls to the ground at the base of the building. How long does the ball remain aloft, and with what speed does it strike the ground?