**B14-1 Final Exam** **December 9, 1993**

1. Compute the following derivatives: (*do not simplify*)

(a) $\dfrac{d}{dx}\ (x^7 - 4x^3 + 2)$

(b) $\dfrac{d}{dx}\ (x^3 + 1)\sin 2x$

(c) $\dfrac{d}{dx}\ \sqrt{\dfrac{2x-1}{3x^2+5}}$

(d) $\dfrac{d}{dx}\ \tan(3x^2 - x + 1)$

(e) $\dfrac{d}{dx}\ \displaystyle\int_0^{\sin x} \sqrt{t^3+1}\ dt$

2. Compute the following integrals:

(a) $\displaystyle\int_0^2 x^3 - x + 1\ dx$

(b) $\displaystyle\int_1^2 \sqrt{2x+1}\ dx$

(c) $\displaystyle\int \frac{x^2 + x - 1}{\sqrt{x}}\ dx$

(d) $\displaystyle\int \frac{\sin x}{(1+\cos x)^5}\ dx$

3. Find the area of the region bounded by $y = x^2$ and $y = 4(x-1)^2$.

4. Given the curve $x^2 - xy + y^2 = 9$,

(a) Find the equation of the tangent line at the point (3,0).

(b) Find where the tangent line is horizontal and where it is vertical.

5. The bottom of a 10 ft ladder, which is leaning against a wall, is being pulled away at a constant rate of 2 ft/sec. At what rate is the top of the ladder descending when the top is 6 ft from the ground?

6. For the function $f(x) = \dfrac{x^2-9}{x^2-4}$, $f'(x) = \dfrac{10x}{(x^2-4)^2}$, and $f''(x) = -10\dfrac{3x^2+4}{(x^2-4)^3}$. Find the following items a) through e) and, finally, sketch the graph of $f(x)$. If there are none of a particular item write *none*. Determine and indicate on the graph:

(a) the asymptotes,

(b) where the function is increasing and where it is decreasing,

(c) where the function is concave upwards and where it is concave downwards,

(d) the inflection points,

(e) the critical points, the local maxima and the local minima.

7. A rectangular box with square base is made of two different materials. The top and bottom each cost 5 cents per square feet while the four sides cost 10 cents per square feet. If the volume must be 4 cu. ft., find the dimensions that minimize the cost. Make sure you fully justify your conclusion that your answer does give a minimum.

8. If a car starts from rest with a constant acceleration of 8ft/sec$^2$, how far has it traveled by the time it reaches a speed of 88 ft/sec?

9. The function $f(x) = x^2 + 3$ has no real roots. Try finding a root anyway by Newton's method starting at $x_0 = 1$. Describe what happens.