

Name: _____

Student ID: _____

TA's name: _____

Mathematics B14-2

FINAL EXAM: ALL SECTIONS

March 13, 1996 — 3:00 to 5:00 p.m.

Please circle your section number.

21 – DiBenedetto,	8	a.m.	57 – Silber,	11	a.m.
31 – DiBenedetto,	9	a.m.	61 – Welland,	12	noon
41 – Christofi,	10	a.m.	77 – Volpert,	1	p.m.
43 – Abdulali	10	a.m.			

Note: *No calculators, notes, books, etc. permitted! Because calculators are not allowed, we expect answers left in a form such as $\sqrt{93}\ln(6) - 100^{\frac{1}{3}}$. Show all of your work for partial credit.*

Do not write below this line

Prob.	Possible points	Score
1	55	
2	15	
3	20	
4	15	
5	30	
6	15	
7	15	
8	20	
9	15	
TOTAL	200	

1. (55) Calculate the following integrals:

(a) (7) $\int \frac{\sin(2x)}{(1 + \cos(2x))^2} dx.$

(b) (7) $\int \sin^{8/3} x \cos^3 x dx.$

(c) (6) $\int \sqrt{x} \ln(x) dx.$

(d) (15) $\int \frac{x^2 + 2}{x^3 + 2x^2 + x} dx.$

(e) (10) $\int \frac{\sqrt{4x^2 - 1}}{x^2} dx.$

(f) (10) $\int_1^{\infty} \frac{1}{x^2 \sqrt{x}} dx.$

2. (15) (a) Calculate $\int x e^{x^2} dx.$

Use the result from (a) together with integration by parts to calculate $\int x^3 e^{x^2} dx.$

3. (20) Differentiate the following functions:

(a) (5) $y = (1 + x)^{2x^2+1}$.

(b) (10) $y = \frac{(3x^2 - 1)(7x + 3)^3}{(4x^2 + 9x + 1)}$.

(c) (15) $y = \arctan\left(\frac{x^2}{2} + \ln x\right)$.

4. (15) Use Simpson's rule to find an estimate for

$$\arctan(2) = \int_0^2 \frac{1}{1+x^2} dx.$$

Break the interval $[0,2]$ into 6 intervals. Show the arithmetic that is to be done, but don't carry out the steps.

5. (30) Consider the region R below the curve $y = 1 - x^2$ and above the x -axis.
- (a) (15) Use the method of discs/washers to calculate the volume of the solid of revolution obtained by revolving R about the line $y = -1$.
- (b) (15) Use the method of cylindrical shells to calculate the volume of the solid of revolution obtained by revolving R about the y axis.
6. (15) Calculate the length of the curve given by $y = \ln(\cos x)$ when $0 \leq x \leq \pi/4$.

7. (15) Consider a bank whose accounts receive 6% interest compounded continuously; that is, if $A(t)$ is the balance at time t then $\frac{dA}{dt} = .06A$. How much money must initially be deposited in order that 18 years later the account holds \$80,000?

8. (20) Find the solution $y = y(x)$ of the differential equation

$$\frac{dy}{dx} = \frac{\cos x}{3y^2} \quad \text{if} \quad y(0) = 1.$$

9. (15) Find the 5th degree Taylor polynomial approximation to the function

$$f(x) = \sin(x) + \cos(x)$$

at $a = 0$.