Math B14-3 Final Exam
June 6, 1990

1. (20 pts) Let $\vec{A} = 7\vec{\imath} + 3\vec{\jmath} - \vec{k}$, $\vec{B} = -\vec{\imath} - 4\vec{\jmath} + 5\vec{k}$, and $\vec{C} = -2\vec{\imath} + 3\vec{\jmath} + 3\vec{k}$. Find the following:

(a) $\vec{A} - 2\vec{B} + 5\vec{C}$

(b) $\vec{A} \cdot \vec{B}$

(c) $\vec{B} \times \vec{C}$

(d) The cosine of the angle between $\vec{A}$ and $\vec{B}$.

(e) $\|\vec{A} - \vec{B}\|$

(f) $\vec{C} \cdot (\vec{A} \times \vec{B})$

(g) The component of $\vec{A}$ along $\vec{C}$, $\quad \mathrm{comp}_{\vec{C}}\,\vec{A}$

2. (15 pts) Find the area of the region inside $r = 2\sin\theta$ and outside the circle $r = 1$.

3. (20 pts) If $\vec{v}(t) = t^3\vec{i} + e^{-t}\vec{j} + \sin t\,\vec{k}$ represents the velocity of a particle at time t, find the following

(a) the postion vector $\vec{r}(t)$, if $\vec{r}(1) = (0, 0, 0)$.

(b) the acceleration vector $\vec{a}(t)$

(c) the speed at time $t = 2$.

(d) $\int_0^\pi \vec{v}(t)dt$

4. (15 pts) Use differentials to approximate $\sin(-0.01)\sqrt{8.98}$.

5. (25 pts) If $f(x, y, z) = e^{xz} + \sin xy + z^2$ evaluate the following

(a) $\dfrac{\partial f}{\partial x}$

(b) $\dfrac{\partial f}{\partial y}$

(c) $\dfrac{\partial f}{\partial z}$

(d) $\dfrac{\partial^2 f}{\partial x \partial z}$

(e) $\dfrac{\partial^2 f}{\partial z^2}$

6. (15 pts) Find the arc length of the following curve: $\vec{r}(t) = t\vec{i} + (\frac{t^3}{6} + \frac{1}{2t})\vec{j}$ for $1 \leq t \leq 3$.

7. (15 pts) Find the equation of the tangent plane to the surface $xy + xz^2 + yz^3 = 1$ at the point $(1, 2, -1)$.

8. (20 pts) Use Lagrange multipliers to find the maximum value of the function $f(x, y, z) = xyz$ subject to the constraints $x + y + z = 30$ and $x - y + z = 0$

9. (20 pts) A building in the shape of a rectangular box is to have a volume of 45,000 cubic feet. Annual heating and cooling cost will amount to \$5 for each square foot of the top, \$1.50 for each square foot for each of the four side walls, and nothing for the bottom. What dimensions of the building will minimize the annual costs?

10. (20 pts) Find the critical points of $f(x, y) = x^3 + y^2 - 6xy + 6x - 2$ and use the second derivative test to check if they are local maxima, local minima, or saddles.

11. (15 pts) Use the chain rule to find $\dfrac{\partial f}{\partial u}$ and $\dfrac{\partial f}{\partial v}$ when $f(x, y, z) = x^2 + \sin y - z$, where $x = u^2$, $y = u + v$, and $z = \ln(v)$