B14-3 Name: ____________________

Final Exam I.D. # ____________________

Spring 1997 Bendel Bernoff Lee Shapiro

Instructions: Write your name and I.D. number above. Circle the name of your instructor. Make sure you have shown all of your work and reasoning on these pages in order to receive full credit. No **books**, **calculators**, or **tables** are allowed. Check that this exam contains 11 pages. Good luck, and have a nice vacation!

	Possible points	**Score**
1	25	
2	15	
3	20	
4	20	
5	25	
6	15	
7	25	
8	20	
9	20	
10	15	
TOTAL	200	

Question 1. Consider a triangle with vertices $A(1,1,1)$, $B(0,1,2)$ and $C(-1,0,-1)$.

a) (8 pts) Find the angle of the triangle at vertex C.

b) (5 pts) Find the area of the triangle ABC.

c) (5 pts) Find the equation of the plane which contains the triangle ABC.

d) (7 pts) Find the parametric equation of the line which passes through B and C.

Question 2.(15 pts) Use the method of differentials to approximate the value of $\sqrt{(5.01)^2 - (3.99)^2}$.

Question 3.(20 pts) Given two numbers such that the sum of their squares is equal to 50, use the method of Lagrange multipliers to find the maximal and minimal value of their sum.

Question 4. Consider the surface given by the equation $3x^2 - y + 4z^2 = 7$.

a) (6 pts) Sketch the level curves of the surface for $z = -2, 0, 1$.

b) (6 pts) Sketch the traces (vertical cross sections) of the surface for $y = -6, 1$.

c) (8 pts) Sketch the surface using a) and b).

Question 5.(25 pts) Decide whether $(0,0)$, $(-1,-1)$, $(1,-1)$ are critical points for the function

$$f(x,y) = x^3 + 3y^3 + 3xy + 3y^2.$$

For each given critical point, determine if it is a relative maximum, relative minimum, saddle point or none of the above. Explain your reasoning.

Question 6.(15 pts) We have a function $g(x, y)$ such that $\dfrac{\partial g}{\partial x} = 2x + y$ and $\dfrac{\partial g}{\partial y} = x + 2y$. Suppose $x = u + v$ and $y = 2u + v$. If we regard g as a function of u and v, what is $\dfrac{\partial g}{\partial u}$ when $u = v = 1$?

Question 7. The equation $\sin(xyz) = z^2$ defines z implicitly as a function of x and y.

a) (15 pts) Determine $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$.

b) (10 pts) What is the tangent plane to the surface $\sin(xyz) = z^2$ at $(1, \pi/2, 1)$?

Question 8. Suppose the surface of a mountain is described by the equation $z = x^3 + y^3$. You are standing at the point $(2, -1, 7)$.

a)(10 pts) What is the slope of incline (rate of change of height with respect to horizontal distance) if you move in the direction of $\mathbf{i} + \mathbf{j}$?

b)(5 pts) Which direction should you proceed (in terms of $\mathbf{i}$ and $\mathbf{j}$) in order have the steepest climb?

c)(5 pts) Suppose you wanted to take it easy on your climb, which direction should you proceed in order for your altitude to remain level?

Question 9. Given a particle which starts from the point (1,1,1) when $t = 0$ and whose velocity is given by $\mathbf{v}(t) = t^2\mathbf{i}+2t^2\mathbf{k}$.

a) (10 pts) Determine the position of the particle when $t = 1$.

b) (10 pts) Determine the length of the trajectory travelled by the particle between $t = 0$ and $t = 1$.

Question 10. (15 pts) Sketch the graph of the equation given by polar coordinates $r = 2\cos\theta; 0 \leq \theta \leq \pi$.