Math B15

Final Exam

Fall 1998

Name: ______________________

I.D. # ______________________

Instructions: Write your name and I.D. number above. **No books, calculators, notes or tables are allowed.** You must show all work on these pages, and make sure that your final answer is clearly shown. If necessary, use the back of the previous sheet to continue your work and clearly indicate the location of your continuation.

Check that this exam contains pages 1–7. Each of the 10 problems counts 20 points: $10 \times 20 = 200$ total points. Good luck, happy holidays and best wishes for 1999.

Circle the name and section of your instructor:

Gasper	9:00
Kitchloo	10:00
Kitchloo	12:00
Trivisa	12:00

Prob.	**Possible points**	**Score**
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
7	20	
8	20	
9	20	
10	20	
TOTAL	200	

1. Evaluate the iterated integral

$$\int_0^{27} \int_{y^{1/3}}^{3} x^{-4} y^2 e^{x^6} \; dx \, dy$$

by interchanging the order of integration.

2. Evaluate the integral

$$\iint_R \frac{\sin(\pi\sqrt{x^2+y^2})}{\sqrt{x^2+y^2}} \, dA,$$

where R is the annular region between the circles $x^2+y^2=1$ and $x^2+y^2=4$.

3. Find the area of the surface of the paraboloid $z = 16 - r^2$ that is above the xy-plane.

4. Find the volume of the solid in the first octant that is bounded above by the surface $\rho = \cos\theta$ and below by the cone $\phi = \pi/4$.

5. Use the substitutions $u = xy$, $v = y/x$ to find the area of the first-quadrant region bounded by the lines $y = x$, $y = 3x$ and the hyperbolas $xy = 1$, $xy = 5$.

6. Consider the vector field $\mathbf{F} = (x + y)\,\mathbf{i} + (x - y)\,\mathbf{j}$.

   (a) Show that $\mathbf{F}$ is conservative and find a function f such that $\mathbf{F} = \nabla f$.

   (b) Evaluate the line integral $\int_C \mathbf{F} \cdot \mathbf{T}\; ds$ in the following two cases:

   (i) C is the positively oriented ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

   (ii) C is the curve given by $\mathbf{r}(t) = (e^t \sin t)\,\mathbf{i} + (e^t \cos t)\,\mathbf{j},\; 0 \le t \le \pi$.

7. Evaluate

$$\oint_C \frac{-y\,dx + x\,dy}{x^2+y^2},$$

where C is the ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1$.

8. (a) Show that the vector field $\mathbf{F} = 3yz\,\mathbf{i} + (3xz - 2yz^2)\,\mathbf{j} + (3xy - 2y^2z)\,\mathbf{k}$ is irrotational.

(b) Find a potential function for $\mathbf{F}$.

9. Use the divergence theorem to evaluate

$$\iint_S \mathbf{F} \cdot \mathbf{n}\, dS,$$

where S that is the surface of the solid within the sphere $x^2 + y^2 + z^2 = 1$ which is bounded below by the xy-plane, $\mathbf{n}$ is the outer unit normal to S, and $\mathbf{F} = 2y^2\,\mathbf{i} + x\,\mathbf{j} + z^2\,\mathbf{k}$.

10. Use Stokes' theorem to evaluate

$$\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, dS,$$

where $\mathbf{F} = -y\,\mathbf{i} + x\,\mathbf{j} + xyz\,\mathbf{k}$, S is the lateral surface of the cone with apex $(0, 0, 10)$ and bottom given by the circle $x^2 + y^2 = 16$, and $\mathbf{n}$ is the outer unit normal to S.