Math B15

Final Exam

Winter 1997

Name: ______________________________

I.D. # ______________________________

Instructions: Write your name and I.D. number above. Show all work on these pages, and make sure that your final answer is clearly shown. No **books**, **calculators**, or **tables** are allowed. Check that this exam contains 10 pages. Good luck, and have a nice holiday!

	Possible points	**Score**
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
TOTAL	100	

1. Compute the integral of $f(x, y, z) = x + y$ over the region bounded by the surfaces $z = 2 - x^2$ and $z = x^2$ for $y \in (0, 3)$.

2. Consider solid region cut out of the solid sphere about the origin and radius 4, by the cylinder $x^2 + (y-1)^2 = 1$. Express its volume in terms of an integral in cylindrical coordinates, but do not evaluate the resulting integral.

3.

3.1 Find the centroid of a homogeneous wire shaped like a semicircle of "radius" R.

3.2 Compute the line integral of $\mathbf{F} \equiv -y\mathbf{i} + x\mathbf{j}$ over the quarter circle of radius 2 about the origin, starting at $(0, 2)$ and going counterclokwise to $(-2, 0)$

3.3 Integrate $f(x, y, z) = xy$ along the elliptical helix of parametric equations $x = 4\cos t$; $y = 9\sin t$; $z = 7t$, for $t \in [0, 5\pi/2]$.

4. Compute the double integrals

$$\int_0^2 \int_{y/2}^1 y e^{x^3}\, dx dy; \qquad \int_0^1 \int_0^{\sqrt{1-x^2}} \left(4 - x^2 - y^2\right)^{-1/2} dy dx.$$

5. Find the mass of the spiral ramp $z = \theta$, $r \in (0, 1)$, $\theta \in (0, \pi)$, given that its density at the generic point of coordinates (r, θ, z) is r.

6.

6.1 Show that the following field is conservative and find its potential.

$$\mathbf{F} = (x + \arctan x)\mathbf{i} + \frac{x+y}{1+y^2}\mathbf{j}.$$

6.2 Find the potential of the conservative field

$$\mathbf{F} \equiv (y\cos z - yze^x)\mathbf{i} + (x\cos z - ze^x)\mathbf{j} - (xy\sin z + ye^x)\mathbf{k}.$$

7. Use Stokes Theorem to evaluate integral of $\mathbf{F} \equiv y^2\mathbf{i} + z^2\mathbf{j} + x^2\mathbf{k}$ along the ellipse intersection of $x + y + z = 1$ with $x^2 + y^2 = 1$, oriented counterclockwise with respect to the poisitve direction of $\mathbf{k}$.

8.

8.1 Find the outward flux of the vector field

$$\mathbf{F} = (x^3 + x \tan z)\mathbf{i} + (y^3 - y \tan z)\mathbf{j} + (z^3 - \arctan xy)\mathbf{k},$$

across the boundary the unit ball.

8.2 Find the flux of $\mathbf{F} \equiv x\mathbf{i} + y\mathbf{j} + 3\mathbf{k}$, across the paraboloid $z = r^2$ for $z \in (0, 4)$. Note that the top is not included.

9. Make use of Stokes Theorem to compute the line integral of $\mathbf{F} = y^2\mathbf{i} + z^2\mathbf{j} + x^2\mathbf{k}$ along the ellipse intersection of $y = z$ and $x^2 + y^2 = 2y$, oriented counterclockwise with respect to the positive direction of $\mathbf{k}$.

10. Compute the integral

$$\iint_S \operatorname{curl}(yz\mathbf{i} - xz\mathbf{j} + z^3\mathbf{k}) \cdot \mathbf{n} dS,$$

where S is the portion of the cone $z = r$ between the planes $z = 0$ and $z = 3$.