E. DiBenedetto, Syllabus for Analysis D-12, Chapters I–VI

(Chapters VII and VIII form a short monographic course)

CHAPTER I. Measure of Sets and Measurable Functions

§ 1. Limits of Sets, Characteristic Functions and σ–Algebras xx
§ 2. Measures xx
2.2. Finite, σ–Finite and Complete Measures
§ 3. Outer Measures xx
§ 4. Constructing Measures from Outer Measures xx
§ 5. Extending Measures from Algebras to σ–Algebras xx
5.1. Necessary and Sufficient Conditions of Measurability xx
§ 6. The Lebesgue Measure of Sets in $I\!R^N$
6.1. A Necessary and Sufficient Condition of Measurability xx
§ 7. The Cantor Set and its Measure xx
§ 8. A Non–Measurable Set xx
§ 9. Vitali Coverings xx
§ 10. Measurable Functions xx
§ 11. The Egorov Theorem xx
11.1. The Egorov Theorem in $I\!R^N$ xx
§ 12. Approximating Measurable Functions by Simple Functions xx
§ 13. Convergence in Measure xx
§ 14. Quasi Continuous Functions and Lusin's Theorem xx
Complements: xx

Compactness and Finite Intersection Property; Completion of a Measure space $\{X, \mathcal{A}, \mu\}$; The Inner Measure and Measurability; The Peano–Jordan Measure of Bounded Sets in $I\!R^N$; On the Pre-Image of a Measurable Set; Borel Sets and Measurable Sets

CHAPTER II. The Lebesgue Integral xx

§ 1. Integral of Simple Functions xx
§ 2. The Lebesgue Integral of Nonnegative Functions xx
§ 3. Convergence Theorems xx
§ 4. Absolute Continuity of the Integral xx

§ 5. The Radon–Nikodym Theorem xx
§ 6. Product of Measures xx
§ 7. On the Structure of $(\mathcal{A} \times \mathcal{B})$ xx
§ 8. The Theorem of Fubini–Tonelli xx
8.1. The Tonelli Version of the Fubini Theorem xx
§ 9. Some Applications of the Fubini–Tonelli Theorem xx
9.1. Integrals in Terms of Distribution Functions xx
9.2. Convolution Integrals xx
9.3. The Marcinkiewicz Integral xx
§ 10. Signed Measures xx
§ 11. Decomposing Measures xx
11.1. The Jordan Decomposition xx
11.2. The Lebesgue Decomposition xx
Complements: xx

Comparing the Lebesgue integral with the Peano–Jordan integral.

CHAPTER III. Topics on Functions of Real Variables xx

§ 1. Functions of Bounded Variations xx
§ 2. Differentiating Functions of Bounded Variation xx
2.1. Differentiating a Series of Monotone Functions xx
§ 3. Absolutely Continuous Functions xx
3.1. Density of a Measurable Set xx
§ 4. Derivatives of Integrals xx
§ 5. Convex Functions xx
5.1. The Jensen's Inequality xx
5.2. The Alexandrov Normal Mapping xx
§ 6. The Tietze Extension Theorem xx
§ 7. The Stone–Weierstrass Theorem xx
§ 8. The Ascoli–Arzelà Theorem xx
Complements: xx

A Nowhere Differentiable Function; The Cantor Ternary Function; A Strictly Monotone Function with a.e. Zero Derivative; The Functions of the Jumps; Convex Functions in $\mathbb{R}^N$; The Legendre Transform; Discrete Versions of Jensen's Inequality. Application to elliptic pde's of the Alexandrov normal mapping.

CHAPTER IV. The $L^p(E)$ SPACES xx

§ 1. Functions in $L^p(E)$ and their Norm xx
§ 2. The Hölder and Minkowski Inequalities xx
§ 3. More on the Spaces $L^p(E)$ and their Norm xx

3.1. Characterizing the Norm $\|f\|_{p,E}$ for $1 \le p < \infty$ xx
3.2. The Norm $\|\cdot\|_{\infty,E}$ for E of Finite Measure xx
3.3. $L^p(E)$ as a Normed Space and Equivalence Functions xx
§ 4. Convergence in $L^p(E)$ and Completeness xx
4.1. Completeness of a Normed Linear Space xx
4.2. The Spaces $L^p(E)$ for $1 \le p \le \infty$ are Complete xx
§ 5. Separating $L^p(E)$ by Step Functions xx
§ 6. Weak Convergence in $L^p(E)$ xx
§ 7. Continuity of the Translation in $L^p(E)$ xx
7.1. A General Version of Proposition 7.1 xx
§ 8. Approximating Functions in $L^p(E)$ with Functions in $C^\infty(E)$ xx
§ 9. Clarkson's Inequalities and Uniform Convexity of $L^p(E)$ xx
9.1. Uniform Convexity of $L^p(E)$ for $1 < p < \infty$ xx
§ 10. Linear Functionals in $L^p(E)$ xx
§ 11. The Riesz Representation Theorem xx
Complements: xx

The spaces ℓ_p; Variants of the Hölder and Minkowski Inequalities; The Continuous Version of Minkowski Inequality; The Reverse Hölder and Minkowski Inequalities; Counterexample to Pointwise and Norm Convergence; Remarks on Weak Convergence; Comparing the Various Notions of Convergence.

CHAPTER V. Further Topics on Functions of Real Variables

This Chapter is in preparation. Toplics include:

Harmonic and parabolic extensions fron $I\!R^N$ to $I\!R^N \times I\!R^+$ and related topics from pde's. The Lebesgue Theorem in $I\!R^N$. Maximal Functions. Operators of Strong type $(p-p)$ and weak type $(1-1)$. Rearrangements.

CHAPTER VI. Metric and Normed Spaces xx

§ 1. Topological Spaces xx
1.1. Hausdorff Topological Spaces xx
1.2. Bases and Axioms of Countability xx
1.3. Product Topologies xx
§ 2. Compact Topological Spaces xx
2.1. Sequentially Compact Topological Spaces xx
2.2. Continuous Functions on Countably Compact Spaces xx
2.3. Product of Compact Spaces xx
§ 3. Vector Spaces xx
3.1. Convex Sets xx
3.2. Linear Maps and Isomorphisms xx
§ 4. Topological Vector Spaces xx

§ 5. Metric and Normed Spaces xx
5.1. Separation and Axioms of Coutability xx
5.2. Equivalent Metrics, Continuity and Products xx
5.3. Metric Vector Spaces and Norma xx
§ 6. Baire Classes and Compact Metric Spaces xx
6.1. Total Boundedness xx
6.2. Coverings and Compactness xx
6.3. Precompact and Totally Bounded Sets xx
§ 7. Partition of Unity xx
7.1. Another Construction of The Partition of Unity xx
§ 8. Precompact sets in $C(\overline{\Omega})$ xx
8.1. Precompact sets in $C(\Omega)$ xx
§ 9. Characterizing Precompact Sets in $L^p(\Omega)$ xx
Complements: xx

The Discrete and the Trivial topologies; The Box Topology; The Alexandrov One–point compactification; Locally Compact spaces. The Hausdorff distance of sets.

CHAPTER VII. Introduction to Sobolev Spaces

§ 1. Distributional Derivatives xx
1.1. Multiindices xx
1.2. Distributions xx
1.3. Examples xx
1.4. Derivatives of Distributions xx
§ 2. Fundamental Solutions xx
2.1. Examples xx
§ 3. Domains xx
3.1. $\partial\Omega$ of Class $C^{1,\lambda}$, and of Class C^1 xx
3.1. $\partial\Omega$ of Class $C^{1,\lambda}$ xx
3.2. Positive Geometric Density on $\partial\Omega$ xx
3.4. The Segment Property xx
3.5. The Uniform Cone Property xx
§ 4. Weak Derivatives Main Properties xx
§ 5. Smooth Approximations xx
5.1. The Meyers–Serrin Theorem xx
5.2. An Extension Lemma xx
§ 7. Duality, Separability and Bases xx
7.1. Separability xx
7.2. basis for $W^{m,2}(\Omega)$ xx
§ 8. The Chain Rule xx
§ 9. Steklov Averagings xx

CHAPTER VIII. Embedding Theorems xx

§ 1. Multiplicative Embeddings of $W_o^{1,p}(\Omega)$ xx
1.1. Proof Of Theorem 1.1 For the Case $(1.2)_i$ xx
1.2. Proof Of Theorem 1.1 For the Case $(1.2)_i$, concluded xx
1.3. Proof Of Theorem 1.1 For the Case $(1.2)_{iii}$ xx
1.4. Proof Of Theorem 1.1 For the Case $(1.2)_{iii}$, concluded xx
§ 2. Potential Estimates xx
2.1. Potential Representations and Limiting Embeddings xx
§ 3. More About Potential Estimates xx
§ 4. Embeddings of $W^{1,p}(\Omega)$ xx
§ 5. Poincaré Inequalities xx
5.1. The Poincaré Inequality xx
5.2. The Discrete Isoperimetric Inequality xx
§ 6. Morrey Spaces and Limiting Embeddings For $p=N$ xx
6.1. The Limiting Case $\alpha = N/p$ xx
§ 7. Compact Embeddings xx
§ 8. Fractional Sobolev Spaces in $\mathbb{R}^N$ xx
§ 9. Traces xx
§ 10. Traces And Fractional Sobolev Spaces xx
§ 11. Traces of Functions In $W^{1,p}(\Omega)$ xx
11.1. Traces and Fractional Sobolev Spaces xx
§ 12. Interpolation Spaces and Inequalities (in preparation) xx