

**D-27 PARTIAL DIFFERENTIAL EQUATIONS**

**Math. Dept. Northwestern Univ.; Winter 2000**

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## PART III

Introduction to elliptic partial differential equations, and calculus of variations. DeGiorgi’s Theorem, Krylov coverings, Harnack estimates.