(1) Let $F$ be a field of cardinality 729. For the following question only, you do not need to justify your answer.
   (a) What is the additive group $F$ as an abstract abelian group?
   (b) What is the multiplicative group $F^\times$ of units of $F$ as an abstract abelian group?
   (c) What is the group of field automorphisms of $F$?

(2) Prove that $\mathbb{Z}[x]$ is a unique factorization domain.

(3) Let $K$ be a field, and let $R = K[x, y]/(y^2 - x^3)$. Prove that the localization of $R$ at $m = (x, y)$ is not a discrete valuation ring.

(4) Let $G$ be a finite group, and let $V$ be a characteristic zero representation of $G$. Suppose that for every $g \in G$, the fixed space $V^g \subset V$ of $v \in V$ such that $gv = v$ has dimension at least $\frac{1}{2} \dim(V)$. Prove that there exists a $v \in V$ such that $gv = v$ for all $g \in G$.

(5) Let $(A, m)$ and $(B, n)$ be local noetherian rings. Suppose that $\phi : A \rightarrow B$ is a map such that $\phi(m) \subset n$, and suppose that:
   (a) $A/m \rightarrow B/n$ is an isomorphism,
   (b) $m \rightarrow n/n^2$ is surjective,
   (c) $B$ is finitely generated as an $A$-module.
   Prove that $\phi$ is surjective.

(6) Let $p$ be prime. Prove that $x^p - x - 1$ is irreducible over $\mathbb{F}_p$. What is the Galois group of its splitting field?