Topology Preliminary Exam **September, 1998**

Do 2 of 3 problems from Part A and 4 of 6 problems from Part B.

Be sure to indicate which problems you are submitting.

Part A

1. Define **chart, atlas** and **manifold.** Define C^∞ **function** from one manifold to another and define the **derivative** of such a map. Give an example of a C^∞ homeomorphism whose inverse fails to have a derivative at some point.

2. (a) Define **transversal intersection** of two submanifolds.
(b) Suppose $A : R^n \to R^n$ is a linear map and let W be the graph of A, i.e. $W = \{(v, A(v)) \in R^n \times R^n \mid v \in R^n\}$. Let $V = R^n \times \{0\} \subset R^n \times R^n$. Prove that V is transverse to W in $R^n \times R^n$ if and only if A is an isomorphism.

3. (a) Define homotopy of two continuous maps.
(b) Suppose $f(t)$ is a loop in (X, x_0), i.e. a continuous function $f : [0,1] \to X$ with $f(0) = f(1) = x_0$. Let $g : [0,1] \to [0,1]$ be a continuous function satisfying $g(0) = 0$ and $g(1) = 1$. Prove that $f(g(t))$ is another loop in (X, x_0), and that it is homotopic to the loop $f(t)$ relative to the base point x_0.

Part B

4. Prove that the boundary of the Möbius Band is a not a retract of the Möbius Band.

5. Let X be the quotient space obtained by identifying the three vertices of a 2-simplex to a single point. Find the homology groups $H_k(X;\mathbf{Z})$ for all k.

6. For each of the following two statements, tell whether the statement is True or False. In each case, support your answer.

(a) There exists a map $f : S^2 \times S^2 \longrightarrow S^4$ such that $f_* : H_4(S^2 \times S^2;\mathbf{Z}) \longrightarrow H_4(S^2;\mathbf{Z})$ is an isomorphism.

(b) There exists a map $g : S^4 \longrightarrow S^2 \times S^2$ such that $g_* : H_4(S^2;\mathbf{Z}) \longrightarrow H_4(S^2 \times S^2;\mathbf{Z})$ is an isomorphism.

7. Show that $\mathbf{R}P^4 \times S^4$ and $\mathbf{R}P^8 \bigvee S^4$ do not have the same homotopy type.

8. Assume that $f : X \longrightarrow Y$ is a covering map, where X and Y are Hausdorff spaces **and** $X = D^2$. Prove that f is a homeomorphism.

9. Let K be the Klein Bottle and let $P = \mathbf{R}P^3$, the 3-dimensional real projective space.
(a) What are $H_k(P \times K;\mathbf{Z})$, for all k?
(b) What are $H^k(P \times K;\mathbf{Z}/2\mathbf{Z})$, for all k?