NORTHWESTERN UNIVERSITY
FIRST SELECTION TEST

Thursday, September 30, 2004

NAME: ______________________________

WILLIAM LOWELL PUTNAM MATHEMATICAL COMPETITION

Problem A1. Find all positive integers x, y such that $4^x + 5 = 9^y$.

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Problem A2. Prove that from any point inside an equilateral triangle, the sum of the measures of the distances to the sides of the triangle is constant.

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Problem A3. Let $a, b, c, d > 0$. Prove that

$$\frac{1}{a} + \frac{1}{b} + \frac{4}{c} + \frac{16}{d} \geq \frac{64}{a+b+c+d}\,.$$

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Problem A4. Find $\lim_{n\to\infty} \prod_{k=0}^{n} \left(1 + \frac{1}{3^{2^k}}\right)$.

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Problem A5. Prove that if a, b are two positive integers and $\sqrt{a}$ is irrational then $\sqrt{a} + \sqrt{b}$ is irrational.

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Problem A6. Prove that in the following product

$$P = (1 - x + x^2 - x^3 + \cdots - x^{99} + x^{100})(1 + x + x^2 + x^3 + \cdots + x^{99} + x^{100})$$

after multiplying and collecting terms, there does not appear a term in x of odd degree.