

ANALYSIS PRELIMINARY EXAMINATION SYLLABUS

1. Real Analysis

- (1) Measurable space and σ -algebra of sets; measurable sets and functions; Borel measurable sets and functions.
- (2) Measure spaces; complete, regular, and σ -finite measures.
- (3) Caratheodory extension theorem; Lebesgue measure on the real line.
- (4) Relation between Borel and Lebesgue measurable sets; non-measurable sets.
- (5) Lebesgue integral of a nonnegative function; integrable functions.
- (6) Monotone convergence theorem; Fatou's lemma; dominated convergence theorem.
- (7) Hölder inequality; Minkowski inequality; Jensen inequality.
- (8) Convergence almost everywhere, convergence in measure, and uniform convergence; relations among the modes of convergence; Egoroff's theorem.
- (9) Signed measures; Hahn and Jordan decompositions; Radon-Nikodym theorem.
- (10) Functions of bounded variation; absolutely continuous functions.
- (11) Differentiability of monotone functions; Lebesgue decomposition theorem; fundamental theorem of calculus.
- (12) Product measure; Tonelli and Fubini theorems.

2. Functional Analysis

- (1) L^p spaces for $1 \leq p \leq \infty$; L^p convergence and completeness.
- (2) Hilbert space; orthonormal basis.
- (3) Fourier series; Riesz-Fischer theorem; representation of bounded linear functionals on a Hilbert space.
- (4) Normed linear space; Banach space; examples of Banach spaces.
- (5) Dual space; Hahn-Banach theorem; embedding in the second dual.
- (6) Riesz representation theorem for L^p with $1 \leq p < \infty$; failure for $p = \infty$.
- (7) Riesz representation theorem for the space of bounded continuous functions on a compact metric space.
- (8) Riesz lemma; noncompactness of the unit ball; Tikhonov compactness theorem; weak and weak*-topologies; Alaoglu theorem.

3. Complex Analysis

- (1) Holomorphic (analytic) function; Cauchy-Riemann equation.
- (2) Power series; convergence radius.
- (3) Cauchy theorem and Cauchy integral formula;
- (4) Meromorphic function; zeros, poles, and essential singularities.
- (5) Local mapping properties and maximum principle for holomorphic functions.
- (6) Liouville theorem; Schwarz lemma; Schwarz reflection principle.
- (7) Residue theorem and application to evaluation of definite integrals.
- (8) Argument principle; Rouché theorem and applications.
- (9) Harmonic function; maximum principle; Poisson kernel and Poisson representation.
- (10) Normal families and Riemann mapping theorem.

Recommended textbooks:

- (1) Ahlfors, Lars, *Complex Analysis*, McGraw-Hill, 3rd edition (1979).
- (2) Conway, John B., *Functions of One Complex Variable*, 2nd edition, Springer-Verlag (1978).
- (3) Conway, John B., *A Course in Functional Analysis*, 2nd edition, Springer-Verlag (1994).
- (4) Folland, Gerald B., *Real Analysis: Modern Techniques and Their Applications*, 2nd edition, Wiley-Interscience (1999).
- (5) Greene Robert E. and Krantz, Steven G., *Function Theory of One Complex Variable*, 3rd edition, American Mathematical Society (2006).
- (6) Royden, Hasley L., *Real Analysis*, 3rd edition, Prentice-Hall (1988).
- (7) Rudin, Walter, *Functional Analysis*, McGraw-Hill, 2nd edition (1991).
- (8) Rudin, Walter, *Real and Complex Analysis*, 3rd edition, McGraw-Hill (1987).
- (9) Stein, Elias M. and Shakarchi, Rami, *Real Analysis*, Princeton University Press (2005).
- (10) Stein, Elias M. and Shakarchi, Rami, *Complex Analysis*, Princeton University Press (2003).