

> Mathematics D70 First-year Graduate Algebra Syllabus

> \*starred topics are optional and will  
> not be covered on the algebra prelim

> GROUPS

> Review of fundamentals [R, ch 1; J, 1.1-1.10]  
> Permutations of sets; Cayley's theorem [R, pp 31-36; C1, p 57]  
> Actions of groups on sets [J, 1.12; R, pp 45-52]  
> Semi-direct products [R, pp 127-138; C2, 4.11]  
> Cosets; Lagrange's theorem; isomorphism theorems  
> Free groups; generators and relations [R, pp 235-242;  
> J, 1.11; C1, 9.9]  
> Normalizers, centralizers, conjugacy classes  
> Class equation  
> Sylow theorems and p-groups [J, 1.13; R, ch 5; L, I.6]  
> Symmetric and alternating groups; simplicity of the latter  
> [R, pp 31-36; C1, p 57]  
> Normal series and composition series  
> \*Jordan-Holder theorem  
> Solvable groups; non-solvability of symmetric groups  
> [R, pp 107-116; L, pp 102-105; C1, pp 219-222 & 230-234]  
> \*Linear groups [R, pp 155-160]  
> \*Nilpotent groups

> RINGS

> Fundamentals (commutativity assumed only when necessary)  
> [J, 2.1-2.7, 3.1-3.3, 3.5; C1, 10.1-10.3;  
> L, II.1, III.1-III.3]  
> units, ideals, homomorphisms, quotient rings,  
> existence of maximal (left, right, 2-sided) ideals,  
> isomorphism theorems, simple rings, 0-divisors,  
> definition of PID, quotient fields of domains  
> Polynomial rings over commutative rings  
> algebraic and transcendental elements, degree, units,  
> division algorithm, minimal polynomials, roots,  
> the multiplicative group of a finite field  
> UFDs; PIDs; Gauss' lemma

> \*Power series rings  
> \*Euclidean rings