

## AND GALOIS THEORY

### Prime fields

Algebraic extensions; algebraic closure

Splitting fields; normal extensions

Separable extensions

Fundamental theorems of Galois theory

Linear independence of characters

Primitive elements

Finite fields

Roots of unity; cyclotomic fields

Wedderburn's theorem on finite division rings

Radical extensions and the insolvability of quintics

\*Transcendental extensions

\*Ruler and compass constructions

### MODULES

#### Fundamentals

Exact sequences; isomorphism theorems; five lemma

Hom and its exactness properties

Products, coproducts and their universal properties

Split exact sequences

Generators; finite generation

Free modules and bases

Projective modules and their relationship to Hom

\*Injective modules

### \*LANGUAGE OF CATEGORIES (whole section optional)

Definitions and examples of categories

Isomorphisms, epimorphisms, monomorphisms

Functors

Natural transformation; the double dual

Products and coproducts

### MATRICES OVER RINGS [L, XIII.1 -XIII.4]

Determinants

Change of basis

Matrix rings over fields: one-sided ideals and simplicity

Application: structure of modules over PIDs [Cl, 10.6]

### MORE RING AND MODULE THEORY (not necessarily commutative)

Chain conditions

Jacobson radical [Rei, 1.6a]

Nilpotent radical

Nakayama's lemma [Rei, 1.6a]