Syllabus – Real and Complex Analysis Prelim

I. Real Analysis

1. Measures (absolutely continuous, Borel, complete, complex, Lebesgue, positive, regular, σ-finite, signed, singular).
2. Relation of Borel measurable sets to Lebesgue measurable sets; a set that is not Lebesgue measurable; a set of Lebesgue measure zero with an uncountable number of points.
3. Monotone convergence theorem, Fatou's lemma, Lebesgue's dominated convergence theorem.
4. L^p-spaces for $1 \leq p \leq \infty$. Completeness, and noncompactness of the unit ball. Hölder, Minkowski, and Jensen inequalities. Young's inequality.
5. Relations between convergence a.e., convergence in measure, convergence in L^p, and uniform convergence. Egoroff's theorem.
6. Radon–Nikodym theorem, Hahn and Jordan decomposition.
7. Riesz representation theorem on L^p, $1 \leq p < \infty$. Failure for $p = \infty$.
8. Riesz representation theorem on the space of bounded continuous functions.
9. Differentiation of measures. Lebesgue decomposition theorem. Hausdorff maximal function. Differentiability a.e. of Lebesgue integrals on the real line.
10. Hilbert space, orthonormal bases, representation of bounded linear functionals on a Hilbert space, Riesz-Fischer theorem for Fourier series.
11. Banach spaces, dual spaces, Hahn-Banach theorem, embedding of a Banach space in its second dual.
12. Fubini's theorem for product measures.

II. Complex Analysis

1. The elementary functions in the complex plane.
2. Power series.
3. Cauchy's theorem and the Cauchy integral formula.
4. Zeros, poles, and singularities of holomorphic and meromorphic functions.
5. Local mapping properties of holomorphic functions.
6. Liouville's theorem, Schwarz's lemma, the Schwarz reflection principle, and the maximum principle for holomorphic functions.
7. The residue theorem and its application to the evaluation of definite integrals.
8. Rouché's theorem, Poisson kernels, and basic properties of harmonic functions including the maximum principle.

References:

W. Rudin, "Real and Complex Analysis," 3rd edition, McGraw–Hill.
H.L. Royden, "Real Analysis," 3rd edition, Macmillan.
J.B. Conway, "Functions of One Complex Variable," 2nd edition, Springer–Verlag.