Dynamical Systems Prelim
September 22, 1988

1. Describe the chain recurrent set of the function $f : R \to R$ given by $f(x) = 100x(1-x)$. Include a description of the dynamics of f restricted to the chain recurrent set and prove that this description is correct.

2. State the following theorems:
   a) The Kupka-Smale theorem
   b) The Shadowing lemma,
   c) Hartman's theorem (also known as the Hartman-Grobman theorem),
   d) The stable manifold theorem for hyperbolic set (include the definitio n of hyperbolic set)

3. Define *structural stability* for a diffeomorphism and state necessary and sufficient conditions for a diffeomorphism to be structurally stable.

4. (a) Define *Markov partition* and *subshift of finite type.*
   (b) Construct a Markov partition for some smooth map of your choice, being sure to give the matrix of the associated subshift of finite type.

5. Let $\sigma : \Sigma_2 \to \Sigma_2$ be the full two shift.
   a) Prove that the periodic points of σ are dense in Σ_2.
   b) Define the *zeta function* of a homeomorphism and calculate the zeta function for the above σ.

6. Let A denote the set of structurally stable diffeomorphisms of the two-sphere S^2 and let B denote the Kupka-Smale diffeomorphisms of S^2. Which of the following are non-empty? For each non-empty set give an example of an element of it (a brief description of the dynamics accompanied by a picture is sufficient).
   a) $A \cap B$
   b) $A^c \cap B$
   c) $A \cap B^c$
   d) $A^c \cap B^c$
   (A^c denotes the complement of A).