Syllabus—Partial Differential Equations Preliminary Examination

(Math 426, Math 427)

(1) Explicit Formulas for Solutions of Certain Linear PDEs

(a) Transport equation: Initial-value problem, nonhomogeneous problem

(b) Laplace's equation: Mean-value formulas, Green's function, energy methods

(c) Heat equation: Fundamental solution, mean-value formula, energy methods

(d) Wave equation: Spherical means, nonhomogeneous problem, energy methods

(2). Sobolev Spaces

(a) Sobolev spaces: weak derivatives, definition of Sobolev spaces

(b) Approximation, extensions, traces

(c) Sobolev Inequalities, Compactness

(d) Hölder spaces, space H^{-1}, spaces involving time

(3). Second-Order Elliptic Equations

(a) Existence of weak solutions, regularity

(b) Maximum principles: weak, strong maximum principles, Harnack's inequality

(c) Eigenvalues and eigenvectors

(4). Linear Evolution Equations

(a) Second-order parabolic equations: Existence, regularity, maximum principles

(b) Second-order hyperbolic equations: Existence, regularity, finite propagation

(c) Hyperbolic systems of first-order equations

(5) First-Order Nonlinear PDEs

(a) Complete integrals, envelopes, characteristics

(c) Hamilton-Jacobi equations: Calculus of variations, Hamiltonian system, Legendre transform, Hopf-Lax formula, viscosity solutions, uniqueness

(d) Conservation laws: Shock, entropy condition, Lax-Oleinik formula, Riemann problem, entropy solutions, uniqueness, long-time behavior

(6). Other Various Ways to Represent Solutions of PDEs

(a) Separation of variables, similarity solutions

(b) Transform methods: Fourier transform, Laplace transform

(e) Converting nonlinear into linear PDEs: Hopf-Cole transformation, potential functions, hodograph and Legendre transform

(f) Power series: Real analytical functions, Cauchy-Kovalevskaya theorem

References:

1. "Partial Differential Equations" by Lawrence C. Evans, Graduate Studies in Math. **19**, Amer. Math. Soc.: Providence, 1998.

2. "Partial Differential Equations" by Fritz John, 4th Edition, Springer-Verlag, New York, 1982.