NORTHWESTERN MASTERCLASS
HEEGAARD FLOER LECTURE SERIES
HOMEWORK 3

(1) Verify that $\partial^2 = 0$ for the toy model of $\widehat{CFD}$ for each of the domains in Figure 1. (I've done the first one for you, so you see what I mean.)

(2) Verify that $\widehat{CFA}$ is a differential module for each of the domains in Figure 2. (Again, I've done the first one for you, so you see what I mean.)

(3) There is a unique pointed matched circle $\mathcal{Z}_1$ for the torus. The corresponding algebra $\mathcal{A}(\mathcal{Z}_1)$ is 8-dimensional (over $\mathbb{F}_2$). Describe it explicitly in terms of generators and relations and/or as a path algebra with relations.

(4) Figure 3 gives three bordered Heegaard diagrams for solid tori. Compute the invariant $\widehat{CFD}(\mathcal{H})$ (which is a differential module over the algebra from Problem 3) for each of these diagrams $\mathcal{H}$.

Remark. Solutions to Problems (1) and (2) can be found in [2], and solutions to Problems 3 and 4 can be found in [1, Section 11.2].

References

[1] Robert Lipshitz, Peter S. Ozsváth, and Dylan P. Thurston, *Bordered Heegaard Floer homology: Invariance and pairing*, 2008, arXiv:0810.0687.

[2] ———, *Slicing planar grid diagrams: a gentle introduction to bordered Heegaard Floer homology*, Proceedings of Gökova Geometry-Topology Conference 2008, Gökova Geometry/Topology Conference (GGT), Gökova, 2009, pp. 91–119, arXiv:0810.0695.

E-mail address: lipshitz@math.columbia.edu

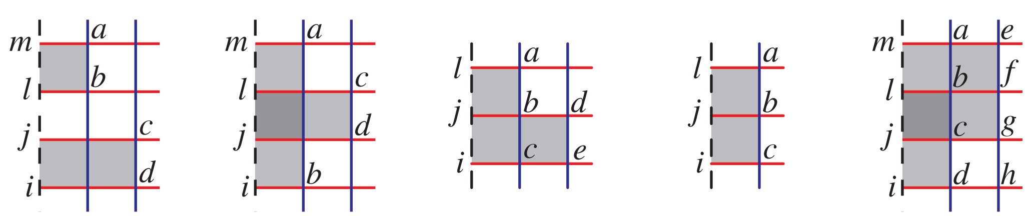

Figure 1. **Examples illustrating $\partial^2 = 0$ for $\widehat{CFD}$.** In part (a), for instance, $\partial^2(\{a,c\}) = \partial(a(\rho_{\ell,m})\{b,c\} + a(\rho_{j,i})\{a,d\}) = a(\rho_{\ell,m})a(\rho_{i,j})\{b,d\} + a(\rho_{i,j})a(\rho_{\ell,m})\{b,d\} = 0$. The darker shading indicates regions involved with multiplicity 2. This figure is drawn from [2].

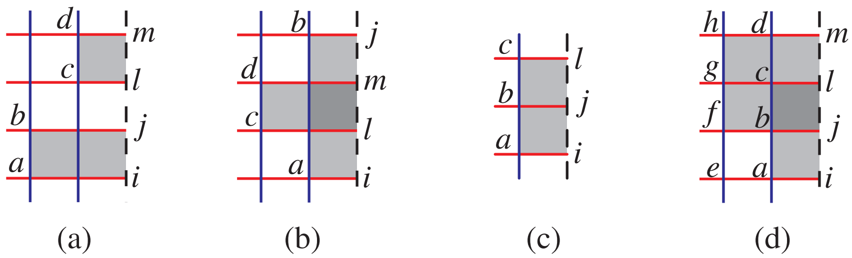

FIGURE 2. **Examples illustrating that $\widehat{CFA}$ respects the relations on the algebra.** In part (a), for instance, $m_2(m_2(\{a,c\},a(\rho_{i,j})),a(\rho_{\ell,m})) = m_2(\{b,c\},a(\rho_{\ell,m})) = \{b,d\} = m_2(\{a,d\},a(\rho_{i,j})) = m_2(m_2(\{a,c\},\rho_{\ell,m}),\rho_{i,j})$. Again, this figure is drawn from [2].

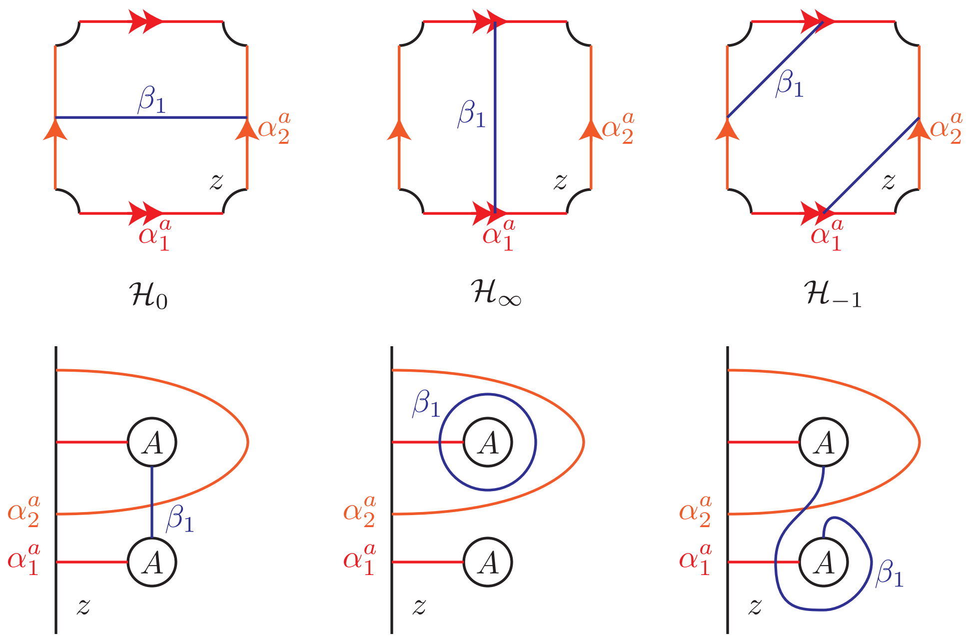

FIGURE 3. **Heegaard diagrams for solid tori.** Each diagram lives on a torus minus a disk, and each diagram is draw in two ways. The arrows indicate edge identifications; the circles labeled by A denote handles; delete the interiors and identify the boundaries of these circles.