
Name:

You have 50 minutes to answer the following 3 questions. Please write all work in the space provided. No calculators, notes or other aids are to be used. Point-values are marked on each problem, for a total of 100. Have fun!

1. (30 points) Determine the values of $c$ for which the set $\{0, c\}$ is a period-2 orbit for $f_c(x) = x^3 - 3x + c$. Is such an orbit attracting for any such value of $c$?

2. (35 points)

(a) Suppose that $f : X \to X$ and that for every $k > 0$, $f$ has exactly $3k$ periodic points of period $k$.

Make a table showing the total number of fixed points of the map $f^k$ for $k \leq 8$. Explain your reasoning carefully.

(Problem 2, continued)

(b) Why can’t there be a function $f : X \to X$ such that for every $k > 0$, $f^k$ has exactly $k^2$ fixed points?

3. (35 points) Let $f_a : \mathbb{R} \to \mathbb{R}$ be given by $f_a(x) = x^3 - ax$.

(a) Find all fixed points and classify them as source, sink, or neither, when $0 < a < 1$.

(Problem 3, continued)

(b) Prove that if $|x|$ is sufficiently large, then $|f_a^n(x)| \to \infty$.

**Suggestion:** Start by showing that if $|x|$ is sufficiently large, then $|f_a(x)| > 10|x|$.