Math 313-2 Test May 6, 2003 R.C. Robinson

No books, no notes, but calculators are allowed.
Show all your work in your bluebook. Start each problem on a new page.

1. (20 Points) Consider the map

$$f(x) = \begin{cases} \frac{3}{2}\,x + \frac{2}{5} & \text{for } 0 \le x \le \frac{2}{5} \\ -\frac{5}{2}\,x + 2 & \text{for } \frac{2}{5} \le x \le \frac{4}{5} \\ 2\,x - \frac{8}{5} & \text{for } \frac{4}{5} \le x \le 1. \end{cases}$$

a. Draw the graph of f. Also, explain why f is an expanding map which has a Markov partition.

b. Give the transition matrix $\mathbf{M} = \left(\dfrac{t_{ij}L_j}{L_i s_i}\right)$ on masses of the subintervals, and find the invariant masses $\mathbf{m}^*$.

c. Find the densities ρ_j^*, which correspond to the invariant masses $\mathbf{m}^*$.

2. (20 Points) Consider the linear map

$$\begin{pmatrix} 2 & 0 \\ 1 & \frac{1}{2} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

from the plane $\mathbb{R}^2$ to itself. Sketch the phase portrait, indicating the stable and unstable manifolds. Also, indicate the behavior of other typical points.

3. (20 Points) Let

$$\mathbf{F}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2\,x\,y + y \\ -x + 3\,y \end{pmatrix}.$$

Find the fixed points and classify them as source, saddle, sink, or none of these.

4. (20 Points) Consider the map given by

$$\mathbf{F}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{cases} \begin{pmatrix} \frac{1}{8}\,x + \frac{1}{8}\,\sin(2\pi\,y) + \frac{1}{8} \\ 8\,y - \frac{1}{2} \end{pmatrix} & \text{for } y < \frac{1}{2} \\[2ex] \begin{pmatrix} -\frac{1}{8}\,x + \frac{7}{8} - \frac{1}{8}\,\sin(2\pi\,(1-y)) \\ -8\,y + \frac{15}{2} \end{pmatrix} & \text{for } \frac{1}{2} \le y. \end{cases}$$

Define the rectangles $\mathbf{R}_0 = [0,1] \times [0, 0.25]$ and $\mathbf{R}_1 = [0,1] \times [0.75, 1]$.

a. Show that $\{\mathbf{R}_0, \mathbf{R}_1\}$ is a Markov partition. Hint: $0 \le \frac{1}{8}\,\sin(2\pi\,y) \le \frac{1}{8}$ for $0 \le y \le 0.25$, and $0 \ge -\frac{1}{8}\,\sin(2\pi\,(1-y)) \ge -\frac{1}{8}$ for $0.75 \le y \le 1$.

b. What is the index of the map from $\mathbf{R}_0$ to itself? From $\mathbf{R}_1$ to itself?

(over)

5. (20 Points) Consider the map given by

$$\mathbf{F}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{cases} \begin{pmatrix} \frac{1}{6}\,x \\ 4\,y \end{pmatrix} & \text{for } -\frac{1}{3} < y < \frac{1}{3} \text{ and } -0.25 \le x \le 1.25 \\ \begin{pmatrix} -\frac{1}{6}\,x + 1 \\ -4\,y + 3 \end{pmatrix} & \text{for } \frac{2}{3} \le y < \frac{4}{3} \text{ and } -0.25 \le x \le 1.25. \end{cases}$$

a. What are the two fixed points?
b. What is the “local” stable and unstable manifolds of the fixed points? Hint: For the “local” stable manifolds, consider the part $-0.5 \le x \le 1.5$ before it leave this region. For “local” unstable manifolds, consider the part $-0.5 \le y \le 1.5$.
c. What is the orbit of period-2?