Since the total surplus is \( \$150,000 - \$135,000 = \$15,000 \), the buyer gets \( 0.99 \times 15,000 = \$14,850 \) of the total surplus of \$15,000 and the seller gets \$150. Thus, after the three-period bargaining process the house sells for \$135,150.

**Problem 7.5.2.** Suppose the bargaining process of the previous problem can go on indefinitely. In this case what is the solution to the bargaining problem?

**Solution:** In case the bargaining continues for an indefinite period, then (according to Theorem 7.30) the equilibrium \( (x^*, 1 - x^*) \) is given by

\[
s^{*\_1,1} = x^* = \frac{1 - 0.99}{1 - 0.99 \times 0.99} = \frac{0.01}{0.0199} \approx 0.50,
\]

and \( s^{*\_1,2} = 1 - x^* \approx 0.50 \).

In this case the solution of the bargaining game splits the total pie exactly halfway between the players. Thus, the buyer and the seller each gets \( 0.5 \times \$15,000 = \$7,500 \) of the surplus and the house sells for \$135,000 + \$7,500 = \$142,500.

**Problem 7.5.3.** In the automobile purchase of Example 7.31, the bargaining takes place between a buyer and a seller with offers and counter-offers. Now suppose that the buyer has the option of going to another dealer who has agreed to sell the Ford Explorer to the individual for \$28,000.

(a) Sketch the bargaining game between the buyer and the seller when the buyer has this "outside option" and the negotiating process is one in which the buyer makes an offer, waits for the counter-offer, and either takes it or rejects it and takes the outside option.

(b) Find a subgame perfect equilibrium of the game described in (a). Is the sale still executed at a price of \$28,000? Explain fully.

**Solution:** (a) This bargaining game is shown in Figure 7.10. The size of the pie in this game is \$3,079 so that if the individual buys the vehicle for \$28,000 then he gets 90 percent of the pie, so that his payoff in case there is no agreement is \( 0.9 \times \delta_1 \). The payoff of the dealer in this case is zero.

(b) The game is now a little different as this is now a two-period game with slightly different payoffs. Clearly, the buyer will accept an offer \( s_{2,1} \) in period 2 if \( \delta_1 s_{2,1} \geq 0.9 \delta_1 \) or, \( s_{2,1} \geq 0.9 \). The seller's payoff in this case is \( \delta_2 (1 - s_{2,1}) \). To maximize his payoff, the seller must offer \( s^*_{2,1} = 0.9 \), in which case he gets \( \delta_2 (1 - s^*_{2,1}) = 0.1 \times \delta_2 \). In the beginning of period 1 player 1 will offer \( s_{1,1} \) such that \( s_{1,2} = 1 - s_{1,1} \geq 0.1 \times \delta_2 \), or \( s_{1,1} \leq 1 - 0.1 \times \delta_2 \). This implies that the subgame perfect equilibrium solution of this bargaining game is given by

\[
s^{*\_1,1} = 1 - s^{*\_1,2} = 1 - 0.1 \times \delta_2, \quad s^{*\_1,2} = 0.1 \times \delta_2 \quad \text{and} \quad s^{*\_2,1} = 0.9.
\]