1. (16 Points) For a collection of lotteries, there are three possible outcomes, $a_1$, $a_2$, and $a_3$. The payoff function satisfies $u(a_1) = 0$, $u(a_2) = v$, and $u(a_3) = 10$. A lottery $(p_1, p_2, p_3)$ indicates a lottery in which the probability of getting outcome $a_i$ is $p_i$. The decision maker prefers the lottery $(0, 0.4, 0.5)$ to $(0.3, 0.7)$ to $(0.275, 0.125, 0.6)$. Are these preferences consistent? And, what can you say about the value of $v = u(a_2)$? Note: $0.125 = \frac{1}{8}$.

2. (16 Points) (Cournot’s duopoly) Because of the form of the costs and inverse demand function, the profit of two firms is given by

$$
\pi_1(q_1, q_2) = q_1 (10 - 2q_2) - 2q_1^2 \quad \text{and} \quad \pi_2(q_1, q_2) = q_2 (8 - 2q_1) - q_2^2,
$$

for production levels $q_1 \geq 0$ by the first firm and $q_2 \geq 0$ by the second firm. Find the Nash equilibrium.

3. Consider the game tree with perfect information given in Figure 1.
   a. (14 Points) What is the subgame perfect Nash equilibrium? Give the strategy profile and payoffs for the players.
   b. (14 Points) Find a strategy profile that is a Nash equilibrium but not subgame perfect. Explain why your choice (i) is not subgame perfect, and (ii) is a Nash equilibrium.

4. (16 Points) In a game of attrition, the payoff functions of two players are given by

$$
u_1(t_1, t_2) = \begin{cases} 
-t_1 & \text{if } t_1 < t_2 \\
5 - t_2 & \text{if } t_1 = t_2 \\
10 - t_2 & \text{if } t_1 > t_2 
\end{cases}
$$

and

$$
u_2(t_1, t_2) = \begin{cases} 
-t_2 & \text{if } t_2 < t_1 \\
6 - t_1 & \text{if } t_2 = t_1 \\
12 - t_1 & \text{if } t_2 > t_1.
\end{cases}
$$

where $t_1 \geq 0$ is the time player one waits and $t_2 \geq 0$ is the time player two waits. What is the best response function of player one, $B_1(t_2)$.

**Hint:** Consider the three cases: (i) $t_2 < 10$, (ii) $t_2 = 10$, and (iii) $t_2 > 10$.

(over for problem 5)
5. Consider the bi-matrix game given by

\[
\begin{pmatrix}
(0, 0) & (8, -4) & (8, -4) \\
(2, -2) & (0, 0) & (2, -2) \\
(1, -1) & (1, -1) & (0, 0)
\end{pmatrix}
\]

a. (8 Points) Are there any pure strategy Nash equilibria? Explain by giving the best response in pure strategies of each player to the pure strategies of the other player.

b. (16 Points) Consider mixed strategies, where player one (who chooses the row) has a strategy \((p_1, p_2, p_3)\) and player two (who chooses the column) has a strategy \((q_1, q_2, q_3)\).

   (i) For \(p_3 = 0 = q_3\) and \(1 \leq i \leq 3\), find the expected payoffs \(E_1(r_i, (q_1, q_2, 0))\) for rows \(r_i\) and \(E_2((p_1, p_2, 0), c_i)\) for columns \(c_i\).

   (ii) Find a mixed strategy Nash equilibrium with \(p_3 = 0 = q_3\). Indicate why it is a Nash equilibrium and determine the payoff for each player.