Math C13: Midterm Exam. Friday, February 13, 1998.

Name:

You have 50 minutes to answer the following 3 questions. Please write all work in the space provided. No calculators, notes or other aids are to be used. Point-values are marked on each problem, for a total of 100. Have fun!

1. (30 points) Determine the values of c for which the set $\{0, c\}$ is a period-2 orbit for $f_c(x) = x^3 - 3x + c$. Is such an orbit attracting for any such value of c?

2. (35 points)

   (a) Suppose that $f : X \to X$ and that for every $k > 0$, f has exactly $3k$ periodic points of period k.

   Make a table showing the total number of fixed points of the map f^k for $k \leq 8$. Explain your reasoning carefully.

   (Problem 2, continued)

   (b) Why *can't* there be a function $f : X \to X$ such that for every $k > 0$, f^k has exactly k^2 fixed points?

3. (35 points) Let $f_a : \mathbf{R} \to \mathbf{R}$ be given by $f_a(x) = x^3 - ax$.

   (a) Find all fixed points and classify them as source, sink, or neither, when $0 < a < 1$.

   (Problem 3, continued)

   (b) Prove that if $|x|$ is sufficiently large, then $|f_a^n(x)| \to \infty$.

   Suggestion: Start by showing that if $|x|$ is sufficiently large, then $|f_a(x)| > 10|x|$.