Math 313-1 Test 2 February 20, 2002

(1) $g(x) = \frac{2}{5}x^3 - \frac{7}{5}x \qquad g'(x) = \frac{6}{5}x^2 - \frac{7}{5}$

$g''(x) = \frac{12}{5}x \qquad g'''(x) = \frac{12}{5}.$

(a)

$$S_g(x) = \frac{\frac{12}{5}\left(\frac{6}{5}x^2 - \frac{7}{5}\right) - \frac{3}{2}\left(\frac{12}{5}x\right)^2}{\left(\frac{6}{5}x^2 - \frac{7}{5}\right)^2}$$

$$= \frac{12x^2\left(6 - \frac{3}{2}\cdot 12\right) - 84}{(6x^2-7)^2} = \frac{-144x^2 - 84}{(6x^2-7)^2} < 0$$

Negative Schwarzian derivative.

(b)

$g'(0) = -\frac{7}{5} \qquad h(0) = \ln\left(\frac{7}{5}\right) > 0$

$g'(\pm\sqrt{6}) = \frac{36-7}{5} = \frac{29}{5} \qquad h(\pm\sqrt{6}) = \ln\left(\frac{29}{5}\right) > 0$

$g'(\pm 1) = \frac{6-7}{5} = -\frac{1}{5} \qquad h(\pm 1) = \frac{\ln\left(\frac{1}{5}\right) + \ln\left(\frac{1}{5}\right)}{2} = -\ln 5 < 0.$

$h\left(\pm\sqrt{\frac{7}{6}}\right) = -\infty$

Any point going thru $\pm\sqrt{\frac{7}{6}}$ will have $h(\;) = -\infty$.
Otherwise get one of the other numbers above.

(c)

$x_0 = g\left(\sqrt{\frac{7}{6}}\right)$ must be in the basin of $\{-1, 1\}$ because periodic sink must have critical point in its basin. Therefore

$h(x_0) = h(\pm 1) = -\ln(5) < 0.$