ERRATA AND ADDITIONS FOR
DYNAMICAL SYSTEMS: STABILITY, SYMBOLIC DYNAMICS, AND CHAOS

BY CLARK ROBINSON

(*) Corrected in second printing.

Also see the list of Corrections for the Second Edition of the book.

p. 20: (L. 8) “x ∈ I” should be “x ∈ I \ {p}”

p. 22: (L. 5-8) This should read as follows: A direct calculation shows that for 2 < µ < 3 the only fixed points of $F^2_µ$ are those for $F_µ$, i.e., 0 and $p_µ$. (See Exercise 2.6.) Since $F^2_µ(1/2)$ is above the diagonal, it follows that $F^2_µ(x)$ is above the diagonal and $x < F^2_µ(x) < p_µ$ for $1/2 ≤ x < p_µ$. Therefore all the points in the interval $[1/2, p_µ]$ converge to $p_µ$ under iteration by $F^2_µ$. Since $|F'_µ(p_µ)| < 1$, it follows that all these points converge to $p_µ$ under iteration by $F_µ$ as well.

p. 22: (L. 7) “below 1/2” should be “below $p_µ$”

p. 23: (L. -18) “implies” should be “imply”

p. 24: (L. 2) “d($f^n(x), ω(x))” should be “d($f^n(x), α(x))”

p. 24: (L. 7) This should read as follows: Similarly, if $f$ is invertible and $y ∈ α(x)$ then $α(y) ⊂ α(x)$ and $ω(y) ⊂ α(x)$.

p. 24: (L. -9) “invariant” should be “positively invariant”

p. 24: (L. -3) “S is closed” should be “S is a closed”

p. 25: (L. 12) This should read as follows: . . . , although there are other . . .

p. 28: (L. -9) remove “expansions have nonunique representations.”

p. 32: (L. -4) “≤ $λ|b - a|$” should be “≥ $λ|b - a|$”

p. 34: (L. 10) . . . derivative of $f$ is always nonzero on the interval $J$, . . .

p. 39: (L. 4) A stronger condition is . . .

p. 39: (L. -9) . . . we describe such a point for $p = 2$. (An obvious change gives the general case.)

Let $t$ be a . . .

p. 39: (L. -3) Therefore $d(σ^n(t), s) ≤ 3^{1-k}2^{-1}$.

p. 40 (L. 10) In the last section we showed

p. 40 (L. -7) “onto” is not needed since it is part of the definition of semi-conjugacy.

p. 42: (L. -8) change “2|z|.” to “2|z|,”

p. 44: (L. 11) “$h ∘ f(x) = g ∘ h$” should be “$h ∘ f(x) = g ∘ h(x)$”

p. 44: (L. -19) “and and” should be “and”

p. 44: (L. -7) $= \lim_{x \to 1} h'_0(x)$. 

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p. 44: (L. -4) Now let \( \{x_i\} \) be an arbitrary sequence with \( x_i \neq 0 \) that . . .
p. 44: (L. -1) "\( h_0 \)" should be "\( h \)

p. 47: (L. 11) "[−2, 0] ∪ [1, 2]." should be "[−2, 0] ∪ [1, 2],"
p. 51: (L. -6) "\( p \geq F^n(0) < p + 1 \)" should be "\( p \leq F^n(0) < p + 1 \)"
p. 55: (L. 7) "\( F^n_{\tau}(0) + m_1 \)" should be "\( F^n_{\tau}(0) + m_2 \)

*p. 57: (2.6c) Prove for any point \( x \)
p. 132: (L. -7 to -4) Replace with: "If \( U \)

p. 141 (Line 7–9) For \( x_0 \in U \) take \( b > 0 \) such that the closed ball \( \bar{B}(x_0, b) \equiv \{x : |x - x_0| \leq b\} \subset U \). The function \( f \) is Lipschitz . . . for all \( x, y \in \bar{B}(x_0, b) \).
p. 179: (Theorem 9.1) The region does not have to be simply connected. It should read: "either an open subset of \( \mathbb{R}^2 \) or \( D = S^2.\)"

*p. 193: (L. 8) "(\( \lambda - c\alpha^{-1} \))^n \geq 2^n" should be "(\( \lambda - c\alpha^{-1} \))^{-n} 2^n"
p. 195: (L. 6) See the separate sheet for an extended addition.
p. 208: (5.40) It should read: “Assume that $\tilde{X}_1(x, a) < 0$, $\tilde{X}_1(x, b) > 0$, and $\tilde{X}_2(x, a) = 0 = \tilde{X}_2(x, b)$ for all $x$.”
p. 215: (L. 10) “include” should be “includes”
p. 215: (L. 9) “directions” should be “subspace”
p. 215: (L. 4) “th” should be “the”
p. 217: (L. -9) It should be “$+2$”
p. 217: (L. -11) It should be “$\cap$”
p. 217: (L. -11) It should be “$\cap$”
p. 218: (L. 1) First sentence should be: Assume $\Lambda$ is a hyperbolic invariant set for $T$
p. 241: (L. 13) “$T_p$” should be “$T_p M$”
p. 242: (L. 1) First sentence should be: Assume $\Lambda$ is a hyperbolic invariant set for $f$ with constants $0 < \lambda < 1$ and $C \geq 1$ giving the hyperbolic structure.
p. 243: (L. -7, -9) These two line should read

$$ \dot{z} = -\frac{8}{3} + xy $$

*p. 236: (L. 16) . . . is called an immersion provided the derivative of $\phi$ at each point is an isomorphism. The image of a one to one immersion is called an immersed submanifold. If an immersion is a homeomorphism then it is called an embedding and its image is called an embedded submanifold. (Some people require that an embedding is also proper, i.e., the inverse image of a compact set is compact.)

*p. 241: (L. 13) “$T_p$” should be “$T_p M$”
p. 241: (L. 15) (iv) there exist $0 < \lambda < 1$ and $C \geq 1$ independent of $p$ such that for all $n \geq 0$ and for all $p \in \Lambda$,
p. 242: (L. 1) First sentence should be: Assume $\Lambda$ is a hyperbolic invariant set for $f$ with constants $0 < \lambda < 1$ and $C \geq 1$ giving the hyperbolic structure.
p. 243: (L. -7, -9) These two line should read

$$ = \bigcup_{n \geq 0} f^{-n}(W^s_{\epsilon}(f^n(p), f)) $$

and

$$ = \bigcup_{n \geq 0} f^n(W^u_{\epsilon}(f^{-n}(p), f)) $$

p. 248 (Line 8-10) In both the definitions of adjacency matrix and transition matrix add the conditions that (ii) $\sum_j a_{ij} \geq 1$ for all $i$, and (iii) $\sum_i a_{ij} \geq 1$ for all $j$.
p. 256: (L. 10) It takes some argument that $\bigcap_{j=n}^{n} F^j(S)$ has $4^n$ components. Any curve $\gamma$ in $S$ from $x = -3$ to $x = 0$ has an image $F(\gamma)$ which reaches in $S$ from $x = -3$ to $x = 3$. Similarly, any curve $\gamma$ in $S$ from $x = 0$ to $x = 3$ has an image $F(\gamma)$ which reaches in $S$ from $x = -3$ to $x = 3$. Applying this to the the top and bottom boundaries of the strips in $F(S) \cap S$ shows that $\bigcap_{j=n}^{n} F^j(S)$ has four strips all the way across $S$ from $x = -3$ to $x = 3$. (There could be other components with just this argument. After proving the existence of invariant cones, it follows there are exactly four components.) By induction, $\bigcap_{j=0}^{n} F^j(S)$ has $2^n$ “horizontal” strips which reach all the way across $S$ from $x = -3$ to $x = 3$. A similar argument applies to $F^{-1}$ to show that $\bigcap_{j=-n}^{n} F^j(S)$ is made up of $2^n$ “vertical” strips which reach all the way across $S$ from $y = -3$ to $y = 3$. A horizontal strip must intersect a vertical
strip in at least one component. Therefore \( \bigcap_{j=-n}^{n} F^j(S) \) has at least \( 4^n \) components. These are nested as \( n \) increases.

p. 256: (L. -11,-10) In the definition of the cones, \( T_p M^n \) should be \( T_p \mathbb{R}^2 \) on both lines.

p. 258: (L. -6) Compare with the argument on the separate extended addition to page 195.

p. 259: (L. -1) A horseshoe is any isolated invariant set which is (i) hyperbolic, (ii) zero topological dimension, and (iii) topologically transitive. By using the existence of a Markov partition, Theorem IX.6.1, a diffeomorphism restricted to such an invariant set is conjugate to a subshift of finite type.

*p. 260: (L. 11) itinerary

p. 262: (L. 4-10) For simplicity below, we take an adapted metric on \( \Lambda_q \). (The adapted metric implies that for \( x \in \Lambda_q \), \( Df_x \) is an immediate contraction on \( E^s_x \) and an immediate expansion on \( E^u_x \).) We extend the splitting \( E^s_x \oplus E^u_x \) on \( \Lambda \) to a continuous (probably noninvariant) splitting \( \hat{E}^s_x \oplus \hat{E}^n_x \) on a (perhaps smaller) neighborhood \( V \) of \( \Lambda_q \). We use cones to show there is an invariant splitting which approximates \( \hat{E}^s_x \oplus \hat{E}^n_x \) and extends the splitting \( E^s_x \oplus E^u_x \) on \( \Lambda_q \). For \( x \in V \), using the adapted metric let

\[
C^s(x) = \{ (\xi, \eta) \in \hat{E}^s_x \oplus \hat{E}^n_x : |\eta| \leq \mu(|\xi|) \}
\]

and

\[
C^u(x) = \{ (\xi, \eta) \in \hat{E}^s_x \oplus \hat{E}^n_x : |\xi| \leq \mu(|\eta|) \}
\]

some \( 0 < \mu < 1 \).

*p. 271: (L. 20) . . . so we want to measure \( \hat{G}(z_0, \epsilon) = H(z^u(z_0, \epsilon)) - H(z^s(z_0, \epsilon)) \). Since \( \hat{G}(z_0, 0) \equiv 0 \), it is possible to write \( \hat{G}(z_0, \epsilon) = \epsilon G(z_0, \epsilon) \). A zero of . . .

p. 298: (Step 2) Compare with the argument on the separate extended addition to page 195.

p. 271: (L. 3b) **Proof** (of Theorem 4.6) We want to apply the Implicit Function Theorem to the function \( G \). We know that \( G(z_0, 0) = M(z_0) = 0 \) and

\[
\frac{\partial G}{\partial \nu}(z_0, 0) = \frac{\partial M}{\partial \nu}(z_0) \neq 0.
\]

Let \( \Sigma \) be a transversal to the flow at \( z_0 \) which contains the direction \( \nu \). Applying the Implicit Function Theorem to \( G \) restricted to \( \Sigma \), there is a function \( z^\ast(\epsilon) \) such that \( G(z^\ast(\epsilon), \epsilon) \equiv 0 \), i.e., \( z^\ast(\epsilon) \) is a homoclinic point for small enough \( \epsilon \). The fact that the homoclinic point is transversal follows from the fact that the derivative is not zero.

p. 291 (L. 15) It should read

\[
= \prod_{k=0}^{n} \det(I - tf_{s_k})^{(-1)^{j+1}},
\]

without the superscript \( j \) on \( f_{s_k} \).

*p. 321: (L 11-12) "\( \hat{W}^s(O(p_j)) \cap \hat{W}^u(O(p_{j+1})) \neq \emptyset \)" should be "\( \hat{W}^u(O(p_j)) \cap \hat{W}^u(O(p_{j+1})) \neq \emptyset \"

*p. 323: (Thm 12.3) "\( \frac{\partial^2 V}{\partial x_i \partial x_j}(x) \)" should be "\( (\frac{\partial^2 V}{\partial x_i \partial x_j}(x)) \)"

p. 327 (7.13) Assume \( A \) is irreducible.

*p. 328: (7.19) This refers to Example 4.1 on page 267, and not the one of page 253.

*p. 328: (7.22) Let \( f_A \) be a hyperbolic toral automorphism on \( \mathbb{T}^n \) with lift \( L_A \) to \( \mathbb{R}^n \). Let \( g \) be a small \( C^1 \) perturbation of \( f_A \) with lift \( G \) to \( \mathbb{R}^n \). Finally, let \( \hat{G} = G - L_A \). Let \( C^0_{b,per}(\mathbb{R}^n) \), \( C^1_{b,per}(\mathbb{R}^n) \), and \( \Theta(G, v) \) be defined as in the proof of Theorem 5.1.
(a) Prove that \( \hat{G} \in C^1_{b,\text{per}}(\mathbb{R}^n) \).
(b) Prove that \( \Theta(\hat{G}, \cdot) \) preserves \( C^0_{b,\text{per}}(\mathbb{R}^n) \).

p. 329: (Improved wording for 7.25) (A horseshoe as a subsystems of a hyperbolic toral automorphism.) Let \( f_{A_2} : T^2 \to T^2 \) be the diffeomorphism induced by the matrix
\[
A_2 = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}
\]
discussed in Example 5.4. Let \( R_{1a} \) be the rectangle used in the Markov partition for this diffeomorphism. Let \( g = f_{A_2}^2 \)
and
\[
\Lambda = \bigcap_{j=-\infty}^{\infty} g^j(R_{1a}).
\]
Prove that \( g : \Lambda \to \Lambda \) is topologically conjugate to the two-sided full two-shift \( \sigma : \Sigma_2 \to \Sigma_2 \). Hint: \( R_{1a} \) plays the role that \( S \) played in the construction of the geometric horseshoe. Prove that \( g(R_{1a}) \cap R_{1a} \) is made up of two disjoint rectangles. (These rectangles are similar to \( V_1 \) and \( V_2 \) in the geometric horseshoe.) Looking at the transition matrix for the Markov partition for \( f_{A_2} \) may help.

p. 335: (Thm 1.2) The only thing that needs to be changed in the proof of Theorem 1.2 is to say “taking the limits in \( n \) and then \( \eta \)”. Note that \( \epsilon \leq \eta \), so as \( \eta \) goes to zero, \( \epsilon \) also must do to zero.

p. 344 (L -9) This should read: “By letting \( \epsilon \) go to zero, \( h_{\text{sep}}(K, f) = h_{\text{span}}(K, f) \).”

p. 360 (Example 4.1:) The map should be
\[
f(t, z) = (g(t), \beta z + \frac{1}{2} e^{2\pi ti})
\]
(a) Prove for \( 0 < \beta < 1/(2\sqrt{2}) \), …

p. 368: (Theorem 1.1) … has a Liapunov function \( L \) such that (i) \( L \) is strictly decreasing off \( \mathcal{R}(\phi^t) \), (ii) if \( L(x) = L(y) \) for \( x, y \in \mathcal{R}(\phi^t) \) then \( x \sim y \), and (iii) \( L(\mathcal{R}(\phi^t)) \) is nowhere dense.

*p. 369: (L 8) “Proposition 1.10” should be “Proposition 1.9”

*p. 369: (L 18-22) (b) The proofs that attracting sets and repelling sets are invariant are similar, so we only look at \( A \). Let \( x \in A \) and fix any real \( s \). Then \( x \in \phi^t(U) \) for all \( t \geq 0 \), so \( \phi^s(x) \in \phi^{t+s}(U) \). Therefore, \( \phi^s(x) \in \bigcap_{t \geq |s|} \phi^{t+s}(U) = A \). Since \( s \) is arbitrary, \( A \) is both positively and negatively invariant.

*p. 370: (L 6) Since \( y \notin U, y \notin A \). From the definition of \( \Omega^+_\epsilon \), it follows that there is a \( T > 0 \) such that \( \phi^T(y) \in U \) so \( \phi^T(y) \notin A^* \). But Proposition 1.2 proves that …

*p. 370: (L -8) Therefore, if \( x \notin A^* \), then \( V(\phi^t(x)) \) goes to zero

p. 372: (Theorem 1.8) … a Liapunov function \( L : M \to \mathbb{R} \) such that (i) \( L \) is strictly decreasing off \( \mathcal{P} \), (ii) if \( L(x) = L(y) \) for \( x, y \in \mathcal{P} \) then \( x \sim y \), and (iii) \( L(\mathcal{P}) \) is nowhere dense.

p. 372: (L 24) If \( x, y \in \mathcal{P} \) and \( x \not\sim y \) then there is a \( k \) such that one of the points is in \( A_k \) and the other is in \( A^*_k \), so \( L_k(x) \neq L_k(y) \) and \( L(x) \neq L(y) \).

p. 380: (L -4) “\( \leq \delta + \epsilon \)” should be “\( \leq \nu + \epsilon \).”

p. 385: (L 8) there is a \( p' \in H_p \) and \( q' \in H_q \) such that …
p. 398: (Theorem 7.1) This should read: \( f : M \to M \) is an Anosov diffeomorphism (\( f \) has a hyperbolic structure on all of \( M \)).

p. 400: (L. -5) \( \ldots \) and so is a conjugacy.

p. 402: (L. 3) is an stable disk at \( x \)

p. 402: (L. 6–9) By construction of the disks and definition of \( F^t \),
\[
(\phi^t(x), \psi^{t(x,h(x))}(h(x))) = F^t(x, h(x)) \in D^u(\phi^t(x), \eta) \cap D^s(\phi^t(x), \eta),
\]
but also
\[
(\phi^t(x), h \circ \phi^t(x)) \in D^u(\phi^t(x), \eta) \cap D^s(\phi^t(x), \eta),
\]
so by uniqueness of this point
\[
h \circ \phi^t(x) = \psi^{t(x,h(x))}(h(x)).
\]

p. 402: (Proposition 8.2) A flow \( \phi^t \) is called flow expansive on \( \Lambda \) provided given \( \epsilon > 0 \), there exists \( \delta > 0 \) such that if \( x_1, x_2 \in \Lambda \) with
\[
d(\phi^t(x_1), \phi^{t(x_2)}) < \delta
\]
for all \( t \) where
\[
\lim_{t \to \pm \infty} \sigma(t) = \pm \infty,
\]
then \( x_2 = \phi^s(x_1) \) for some \( |s| \leq \epsilon \).


p. 408: (L. 5)
\[
"d(D^{-i}(x_j), x_{j-i}) \leq \delta(\frac{1}{q} + \cdots + \frac{1}{q^j}) \leq \delta"
\]
should be
\[
"d(D^{-i}(x_j), x_{j-i}) \leq \delta(\frac{1}{2} + \cdots + \frac{1}{2^n}) \leq \delta."
\]

p. 409: (Problem 22) It is necessary to assume that all points are nonwandering. (Problem 22b)
Prove that \( W^s(q) \) is dense in \( M \).

p. 427: (L. 2-3) These should be 
\( -\hat{f}(x)D(\beta_r)_x \) not 
\( +\hat{f}(x)D(\beta_r)_x \)

p. 427: (L. -3) \ldots the transversality condition with respect to \( \mathcal{R}(f) \).

p. 428 (L. 3-5) Assume \( f \) is \( C^1 \) structurally stable. Then by Theorem 4.3 (or Exercise 10.11(c)),
it then follows that \( f \) also satisfies the transversality condition with respect to \( \mathcal{R}(f) \).

p. 434: (L. -7) \ldots complete spaces (closed balls in normed linear spaces) since \ldots

p. 438: (L. -3)
\[
\mathcal{L}(C^u) = \sup \{ \frac{|w - w'|}{|v|} : (v, w), (v, w') \in C^u \subset T_x X \times Y \}.
\]

p. 439: (L. -9) Compare with the argument on the separate extended addition to page 195.

p. 443: (L. 18) "Since the invariant section has \( E^u_x \) as a graph, the unstable bundle is \( C^1 \)," should be "By uniqueness, the invariant section has \( E^u_x \) as a graph and the unstable bundle is \( C^1 \)."
The following people were some of those who told me about errata in the book: Youngna Choi, James Jacklitch, Ming-Chia Li, Jody Sorensen, and Dick Swanson.

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