1. Let \( \text{char}(F) = p \). If \( p = 0 \), show that \( \dim(H_n(X, \mathbb{Z})) = \dim(H_n(X, F)) \) for each \( n \) by using the universal coefficient theorem. If \( p > 0 \), show that \( \dim(H_n(X, F)) = \dim(H_n(X, \mathbb{Z})) + a_n + a_{n-1} \), where \( a_n \) is the dimension of \( \mathbb{Z}/p\mathbb{Z} \) in the torsion part of \( H_n(X, \mathbb{Z}) \). The alternating sum cancels out \( a_n + a_{n-1} \).

2. \( n = 0 \) case is easily verified. For \( n > 0 \) case, \( H^n(X, F) = \text{Hom}(H_n(X, \mathbb{Z}), F) \oplus \text{Ext}(H_{n-1}(X, \mathbb{Z}), F) \). By assumption, for \( F = \mathbb{Z}/p\mathbb{Z} \) for \( \mathbb{Q} \), the right hand side is zero. In particular, the first component is zero. If \( F = \mathbb{Q} \), we find that the rank of \( H_n(X, \mathbb{Z}) = 0 \). For \( F = \mathbb{Z}/p\mathbb{Z} \), we find the rank of the \( p \)-torsion part of \( H_n(X, \mathbb{Z}) \) is zero. Thus \( H_n(X, \mathbb{Z}) = 0 \) by the structure theorem of finitely generated abelian groups.