1. Notice that $\mathbb{T}^n = \mathbb{T}^{n-1} \times S^1$. Apply Kunneth formula and prove that $H^k(\mathbb{T}^n) = \binom{n}{k}$ by induction.

2. Apply Kunneth formula for $H^k(M \times N)$. Then use algebra to complete the proof (use definition of $\chi(M)$).

3. By Poincare duality, $dim(H^k(M)) = dim(H^{n-k}(M))$ where $n = dim(M)$. If $n$ is odd, the alternating sum is zero (everything cancel out).