CS 310
Homework Assignment No. 2
Due on Tue 4/13/2005

1. In a university with 300 students enrolled 140 students are taking French, 120 are taking business, 130 are taking music, 30 are taking French and business, 40 are taking business and music, 50 are taking French and music, and 10 are taking French, business and music. How many students:
(a) are taking exactly two of those subjects?
(b) are taking exactly one of those subjects?
(c) are not taking any of the three subjects?

2. For each of the following functions determine whether it is one-to-one, or onto, or both. If the function is bijective find its inverse.
(a) $f : \mathbf{Z} \to \mathbf{Z}$, $f(n) = 2n + 1$.
(b) $f : \mathbf{Z} \to \mathbf{Z}$, $f(n) = 7 - n$.
(c) $f : \mathbf{Z} \to \mathbf{Z}$, $f(n) = \lfloor \frac{n+1}{2} \rfloor$.
(d) $f : \mathbf{Z} \times \mathbf{Z} \to \mathbf{Z}$, $f(m, n) = m + n$.
(e) $f : \mathbf{Z} \times \mathbf{Z} \to \mathbf{Z}$, $f(m, n) = m^2 + n^2$.
(f) $f : \mathbf{Z} \to \mathbf{Z} \times \mathbf{Z}$, $f(n) = (n^2, n^2)$.
(g) $f : \mathbf{Z} \to \mathbf{Z} \times \mathbf{Z}$, $f(n) = (n^2, n^3)$.
(h) $f : (0, 1) \to \mathbf{R}^+$, $f(x) = \dfrac{1}{1 - x} - 1$.

3. Let f and g be the functions from $\mathbb{Z}^+$ to $\mathbb{Z}^+$ ($\mathbb{Z}^+$= positive integers) defined by $f(n) = n^2$, $g(n) = 2^n$. Find the compositions $f \circ f$, $g \circ g$, $f \circ g$, and $g \circ f$.

4. Find the properties (reflexive, transitive, symmetric, antisymmetric) verified by the following relations:
(a) Strict inequality of integers: $x \,\mathcal{R}\, y \Leftrightarrow x < y$.
(b) Set disjointness: $A \,\mathcal{R}\, B \Leftrightarrow A \cap B = \emptyset$.
(c) The following relation on $\mathbb{Q}$: $x \,\mathcal{R}\, y \Leftrightarrow x - y \in \mathbb{Z}$.
(d) The following relation on $\mathbb{Q}$: $x \,\mathcal{R}\, y \Leftrightarrow x - y \in \mathbb{N}$.

5. Prove that the following is an *equivalence relation* on $\mathbb{Z} \times \mathbb{Z}^*$:

$$(a, b) \,\mathcal{R}\, (c, d) \Leftrightarrow a\, d = b\, c\,.$$

Is it an equivalence relation on $\mathbb{Z} \times \mathbb{Z}$?

6. Draw the Hasse diagrams for the following relations:
(a) Inclusion on the set $\mathcal{P}(S)$, where $S = \{a, b, c\}$.
(b) Divisibility on the set of all positive divisors of 30.
(c) Divisibility on the set of all positive divisors of 24.