1. (Inequalities) Find the minimum value of the function
   \[ f(x, y, z) = \frac{x}{y} + \frac{y}{z} + \frac{z}{x} \]
   for \( x, y, z > 0 \).

2. (Algebra) Express \( F = \frac{\sqrt[3]{2}}{1 + \sqrt[3]{2} + \sqrt[3]{4}} \) with a rational denominator.

3. (Polynomials) Prove that \( (2 + \sqrt{5})^{1/3} + (2 - \sqrt{5})^{1/3} \) is rational.

4. (Number Theory) Find all prime numbers of the form \( n^n + 1 \) which are less than \( 10^{19} \).

5. (Induction) Prove that \( 3^{n+1} \) divides \( 2^{3^n} + 1 \) for all integers \( n \geq 0 \).

6. (Recurrences) Determine the maximum number of regions in the plane that are determined by \( n \) “vee”s. A “vee” is two rays which meet at a point. The angle between them is any positive number.

7. (Telescoping) Find the infinite product \( \prod_{n=0}^{\infty} \left( 1 + \frac{1}{3^{2n}} \right) \).

8. (Pigeonhole Principle) (IMO 1972.) Prove that from ten distinct two-digit numbers, one can always choose two disjoint nonempty subsets, so that their elements have the same sum.

9. (Generating Functions) Find the infinite sum \( \sum_{n=1}^{\infty} \frac{n}{2^n} \).
10. (Symmetries) (Putnam 1980) Evaluate \[ \int_{0}^{\pi/2} \frac{dx}{1 + (\tan x)\sqrt{2}}. \]

11. (Combinatorics) A parking lot for compact cars has 12 adjacent spaces, and 8 are occupied. A large sport-utility vehicle arrives, needing 2 adjacent open spaces. What is the probability that it will be able to park?

12. (Complex Numbers) Find a close-form expression for \( \prod_{k=1}^{n-1} \sin \frac{k\pi}{n}. \)

13. (Calculus) Compute \( \lim_{n \to \infty} \left\{ \prod_{k=1}^{n} \left( 1 + \frac{k}{n} \right) \right\}^{1/n}. \)

14. (Games) Consider the following two-player game. Each player takes turns placing a penny on the surface of a rectangular table. No penny can touch a penny which is already on the table. The table starts out with no pennies. The last player who makes a legal move wins. Does the first player have a winning strategy?

15. (Other) Let \( f : \mathbb{R} \to \mathbb{R} \) be a continuous function such that \( f \circ f \) has a fixed point, i.e., there is some real number \( x_0 \) such that \( f(f(x_0)) = x_0 \). Prove that \( f \) also has a fixed point.