Due Wednesday, October 22

(1) [Ha] Ch.I: 2.9, 2.12, 2.14, 2.17 (part (c) challenge, and for part (d) you get 
an automatic Ph.D.), 3.14

(2) A conic in $\mathbb{P}^2$ is a projective variety that can be written as the zero locus of 
a single irreducible homogeneous polynomial of degree 2. Let $V$ be the 
vector space over $k$ of homogeneous degree 2 polynomials in three variables, 
and let $\mathbb{P}(V) \cong \mathbb{P}^5$ be its projectivization.

(a) Show that the space of conics in $\mathbb{P}^2$ can be identified with an open 
subset $U \subset \mathbb{P}^5$. (We say that $U$ is a moduli space for conics in $\mathbb{P}^2$ 
and that $\mathbb{P}^5$ is its compactification.) What geometric objects can be 
associated with the points in $\mathbb{P}^5 - U$?

(b) Show that the condition for a conic to pass through a given point is 
a linear one; namely, if $P \in \mathbb{P}^2$, show that there is a linear subspace 
$L \subset \mathbb{P}^5$ such that the conics passing through $P$ are exactly those in 
$U \cap L$. What do the points in $(\mathbb{P}^5 - U) \cap L$ correspond to?

(c) Show that there is a unique conic passing through any five points in 
$\mathbb{P}^2$, as long as no three of them lie on a line. What happens if three of 
them do lie on a line?

(3) Show that a morphism of affine varieties $f : X \to Y$ is dominant if and 
only if $f^* : A(Y) \to A(X)$ is injective.

(4) [Ha] Ch.I: 4.5, 4.6, 4.10